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Modelling of Ventilation: Diving Apparatus

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ABSTRACT

The result of the research carried out and described in this cycle of articles is the development and verification of the approach to modelling of the ventilation processes. The presented approach is a return to deterministic modelling, abandoned due to the lack of good methods to assess the volume of ventilated space. Finding a way to measure this value was the key element in the carried-out research. Search for mathematical models is a commonly used procedure. When dealing with complex issues, methods to search for empirical or semi-empirical models are used. With regard to less complicated problems, attempts are made to develop analytical models that have physical interpretation. The presented approach can completely change the current approach to ventilation processes by replacing semi-empirical models with more accurate analytical models. Validating the latter is often unnecessary because they are based on the verified laws of nature. The research carried out was aimed at developing and verifying a general analytical mathematical model of the ventilation process, both for objects with semi-closed and closed circulation of breathing gas, for hyperbaric and norm baric conditions.

This cycle of articles will be useful for engineers designing devices in which ventilation processes take place, in particular designers: breathing apparatus, submarines, mines, safety chambers, industrial buildings and other closed or semi-closed compartments.

Keywords: Ventilation; Diving Apparatus; Decompression Chamber; Mining Cave; Submarine

Introduction

The mathematical models of ventilation in diving apparatuses have been presented in the cycle of articles deal with ventilation of the hyperbaric chambers, the submarines and the mining excavation. The mathematical models of ventilation for Semi-Closed-Circuit Rebreather (SCR) are based on the mass balance of oxygen in a respiratory mixture inhaled by a diver. For time approaching infinity t→∞, the mathematical models proposed here became the same as the models proposed by the other authors (Haux G [1]). The full time-dependent models are useful during design of the diving apparatus and suitable decompression. In many cases they are good enough to be useful for illustrating the important phenomena occurring in the diving apparatus. Frequently conclusions concerning the composition stabilization rate of the diving apparatus atmosphere following disturbances caused by a change in the oxygen consumption level or caused by periodical ventilation can be drawn based on that model (Kłos R [2,3]). The most important problem associated with the modelling of ventilation refers to a semi-closed rebreather SCR. In SCRs a relative decrease in the oxygen concentration in the inhaled breathing medium is observed compared to the oxygen content in the stored fresh mixture or gas mixture dynamically prepared from gases stored in bottles integrated with the SCRs. The results of the research carried out and described in this cycle of articles are the development and verification of the approach to modelling of the ventilation processes.

The presented approach is a return to deterministic modelling, abandoned due to the lack of good methods to assess the volume of ventilated space. Finding a way to measure this value was the key element in the carried-out research. The classical models are not precise enough as regards the presented here introduction of Semi-closed Circuit Self-container Underwater Breathing Apparatus CRABE - SCR CRABE SCUBA (Kłos R [3-7]). The experiments concerning the mathematical ventilation models were performed during manned and unmanned experimental diving aimed at verification of the decompression procedure proposed. The experiments were carried out for semi-closed rebreathers SCRs. The tested SCRs should feature the same parameters as the parameters assumed to derive the mathematical model. Traditionally the ventilation modelling assessment has been conducted using semi-empirical methods like dimensional analysis (Kłos R [8]). The classic approach is based on experiments with physical models in increased or decreased scale. The experiments concerning the mathematical ventilation models were performed during manned and unmanned experimental diving through more than 40 years (Kłos R [9,10]).

Materials and Methods

Implementation of newly developed structural solutions applied in SCR is inseparable with carrying out experiments involving people who are exposed to risks associated with the inherent necessity to test new or modernized diving technologies. Firstly, in the new cycle of scientific investigations there were performed the unmanned experiments by means of a combined respiratory, metabolic and hyperbaric simulator. Its use should reduce risk in such research. The aim of the design should be to develop a pressure attachment that enables oxygen uptake from the breathing atmosphere, then humidifying and replacing oxygen taken from the atmosphere by a suitable amount of carbon dioxide emitted. Solution to this problem will enable modelling the processes occurring in SCR. This will enable saving time, costs of diving equipment design and costs of developing decompression schedules. As there was no verified mathematical model available, it was necessary to reconstruct the SCR prototype after each series of the preliminary investigations. Moreover, the model to be and verified model will make it possible to evaluate the diving systems in use. To simulate the respiratory process a mechanical representation of pulmonary ventilation and gas exchange in the course of breathing are needed. The processes should be precisely and repeatedly represented, however there is no need for each detail of the real respiratory cycle is to be represented. Usually, it is assumed that an approximated shape of respiration should be sinusoidal.

When the body gas exchange is simulated, attention should be focused on oxygen consumption and its replacement by the carbon dioxide emitted. A respiratory quotient defines the volumetric ratio for the emitted carbon dioxide that replaces the oxygen consumed.

Construction

The CRABE (Complete Range Autonomous Breathing Equipment) is a SCR with two breathing bags placed one inside the other. It is powered by gas mixture as premix or pure oxygen. The proper circulation of the breathing mixture is maintained by the directional valves 1 – Figure 1. During the expiration phase, the exhaust valve of the mouthpiece opens – Figure 1A. The breathing mixture exhaled through the mouthpiece 12, the exhalation hose, and the exhaust valve, passes through a carbon dioxide scrubber 11 into to the large breathing bag 2, and from there through the non-return valve into the small breathing bag 2. When the diver inhales, the exhaust valve closes, and the inlet valve opens – Figure 1B. The breathing mixture is inhaled into the lungs from the large bag 2, through the inlet valve, inhalation hose and mouthpiece 12. This causes the large bag 2 to shrink, along with the small bag 3, from which the gas escapes through a relief valve 4 to ambient water. As the large bag 2 is shrinking, it triggers the dosing valve 5 by pressing its lever. Opening the valve 5 causes fresh breathing mixture to be inhaled from cylinder 10 through the valve 9, coupler 8, regulator valve 6, and dosing valve 5 to the large bag 2, where it is mixed with regenerated breathing mixture. From here, the breathing mixture is inhaled by the diver through the non-return valve, inhalation hose and mouthpiece 12. The demand for breathing mixture is regulated by breathing and the ratio of the volume of the large bag to the small one.

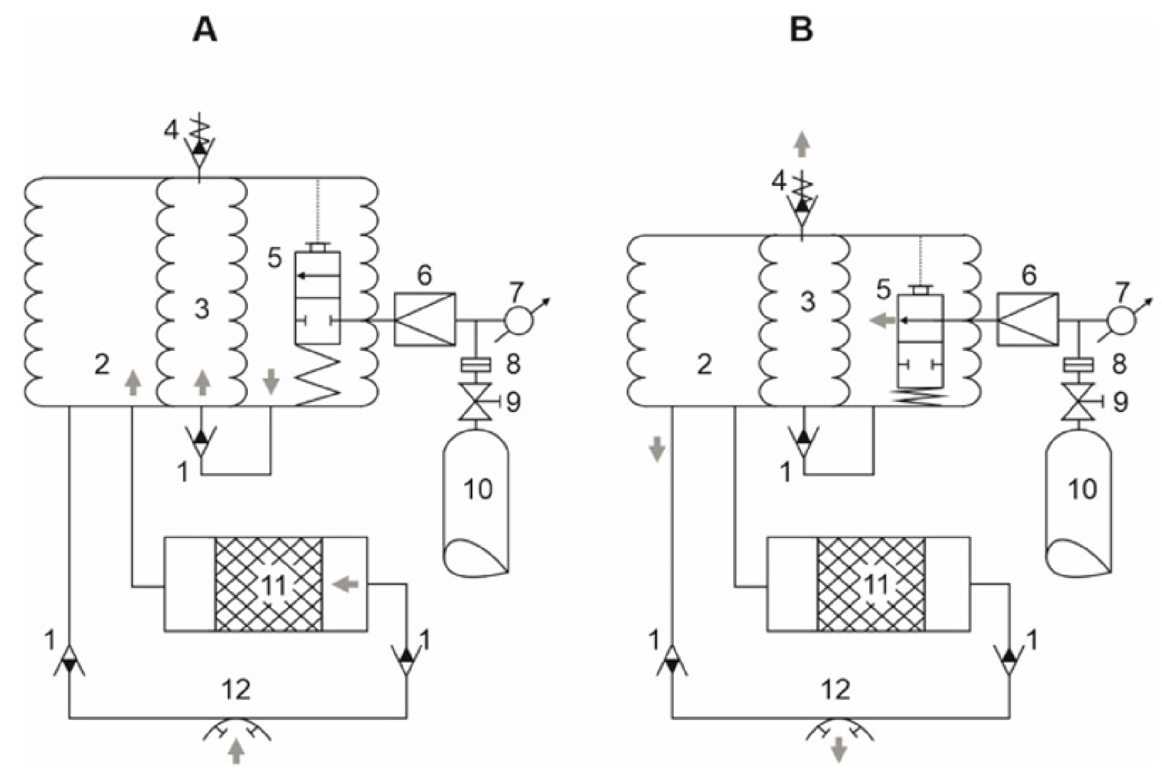

Figure 1: Principle of operation of a semi-closed circuit rebreather with breathing bags placed one inside the other, where:

A. Expiration phase,
B. Inspiration phase,
1. Non-return valve,
2. External large bag,
3. Inner small bag,
4. Relief valve,
5. Dosing valve,
6. Regulator valve,
7. Manometer,
8. Coupling,
9. Shut-off valve,
10. Cylinder with breathing mixture,
11. Carbon dioxide scrubber,
12. Mouthpiece

Ventilation Model

In a SCR construction with a metering bellows dispenser, like previously, there is a relative decrease in the oxygen content to the stable value x_s in the breathing space in relation to the oxygen content in the premix delivered to this space x_w. The breathing gas that remains in circulation is always a breathing mixture, because even during oxygen decompression the diver does not breathe pure oxygen as in the circuit there is always some circulating nitrogen N_2 and/or helium He left from the breathing gas, which comes from places that are not ventilated sufficiently in the breathing space of the apparatus and from the diver's tissue wash out with oxygen. The assessment of the effectiveness of cleansing of the breathing space and the diver's body with oxygen is crucial for the correct design of safe decompression. During inspiration part of the breathing mixture is taken: $-V_1$. The elementary volume of inspiration $-dv_I = -dh \cdot (A - a)$ will be the product of the elementary change in the height of bags -dh multiplied by the area of the base of the large bag A minus the area of the small bag's base area

-a - Figure 2. At this time, the elementary drop in the height of bags $-dh$ will cause a release of the content of the small bag dv_u to water, equal to the product of the elementary drop in height $-dh$ and surface area of the small bag a: $-dv_u = -dh \cdot a$. The ratio of elementary volume of the gas released from the small sack to water $-dv_u$ to the elementary volume of inspiration $-dv_1$ will be: $\frac{-dv_u}{-dv_1} = \frac{a}{A-a} = \frac{u}{U-u} \equiv r$, where u represents the volume of the small bag and U the volume of the large bag. It follows from the above considerations that the released number of moles of the breathing mixture, and as a consequence of oxygen $-n_1 = f\left(\dot{V}_E, r\right)$, is associated with lung ventilation $\dot{V}_E$ by means of the volume ratio of small bag u and large bag U: $r = \frac{u}{U-u}$. Thus, the molar stream of oxygen escaping through the small bag relief valve $-\dot{n}_1$ can be estimated using the *Clapeyron equation* as: $-\dot{n}_1 = -\frac{p}{R.T} \cdot \dot{V}_E \cdot r \cdot x$, where p represents the pressure at the diving depth H: $P = f(H)$.The same number of moles $-\dot{n}_1 = -\frac{p}{R.T} \cdot \dot{V}_E \cdot r \cdot x$ decreases during inspiration from the large bag to the small bag Figure 2.

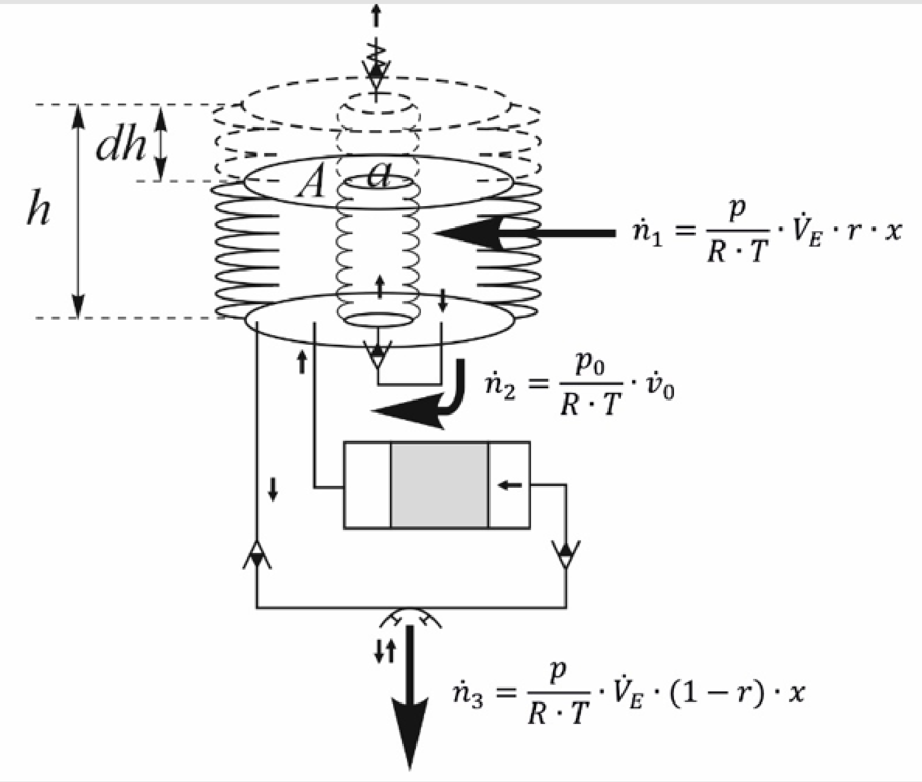

Figure 2: Diagram of operation of the bag system in the bag during inspiration, where: A-area of the base of the large bag, a-area of the base of the small bag, dh-elementary change in height of bags, h-standard height of bags, p-pressure the diving depth, p_0 -normal pressure, R-universal constant of gas, T- absolute temperature, U-volume of the large bag, u-volume of the small bag, r-volume ratio of the small bag to the large bag $r = \frac{u}{U-u}$, $\dot{V}_E$ -ventilation of lungs, $\dot{v}_0$ -stream of consumed oxygen, x-molar fraction of oxygen in the fresh breathing gas, x_w -molar fraction of oxygen in the fresh breathing gas, x_0 -molar fraction of oxygen in the breathing space before the diving apparatus is started.

During inspiration, the oxygen stream having value $-\dot{n}_2 = -\frac{p_0}{R.T} \cdot \dot{V}_0$ is consumed from the breathing mixture drawn in. In the same phase the dosing system resupplies the volume of the large bag with fresh premix, delivering oxygen having the value: $-\dot{n}_3 = \left(\frac{P}{R.T}.\dot{V}_E.r + \frac{P_0}{P}.\dot{v}_0\right).x_w$. Thus, the elementary change in oxygen content ∂x in volume of the large bag (U-u) at time ∂t can be written as: $\frac{P}{R.T}.(U-u).\frac{\partial x}{\partial t} = \dot{n}_3 - \dot{n}_2 - \dot{n}_1$ Figure 2. After setting the values of streams, the ordinary first order differential equation can be written:

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$$\frac{P}{R\cdot T}\cdot(U-u)\cdot\frac{\partial x}{\partial t}=\frac{P}{R\cdot T}\cdot\left(\dot{V}_E\cdot r+\frac{P_0}{P}\cdot\dot{V}_0\right)\cdot x_w-\frac{P_0}{R\cdot T}\cdot\dot{V}_0-\frac{P}{R.T}\cdot\dot{V}_E\cdot r\cdot x \quad (1)$$

where: $\frac{\partial x}{\partial t}$ - elementary change in oxygen content ∂x in the large bag for elementary time change ∂t, other designations as in Figure 2.

The solution of Equation 1 in the form of mathematical proof is presented in Table 1:

$$x(t)=x_w-\varepsilon_p\cdot\varepsilon\cdot\varepsilon_k+\left(x_0-x_w+\varepsilon_p\cdot\varepsilon\cdot\varepsilon_k\right)\cdot exp\left(-\frac{\dot{V}_0.r^2}{\varepsilon_0\cdot u}\cdot t\right) \quad (2)$$

where: x_0- molar fraction of oxygen in the breathing space before the diving apparatus is started, $\varepsilon_K=\frac{1-x_w}{r}$-module representing the structural solution of the breathing space in the diving apparatus, $\varepsilon=\frac{\dot{v}}{\dot{V}_E}$- breathing module represents ventilatory equivalent for oxygen $V_E(O_2)$, $\varepsilon_p=\frac{p_0}{p}$ - *pressure module*; here the module is referred to as a formally dimensionless quantity; the modules will also be referred to as *criterion numbers*.

Table 1: Derivation of the ventilation model related to oxygen content x in function of time $x=f(t)$ for a semi-closed-circuit rebreather with a system of bags placed in above another.

	$\dot{v}_0 \neq f(H)$ a) $x \to x = f(t)$ b)	
1	$\frac{p}{R\cdot T}\cdot(U-u)\cdot\frac{\partial x}{\partial t}=\frac{p}{R\cdot T}\cdot\left(\dot{V}_E\cdot r+\frac{p_0}{p}\cdot\dot{v}_0\right)\cdot x_w-\frac{p_0}{R\cdot T}\cdot\dot{v}_0-\frac{p}{R\cdot T}\cdot\dot{V}_E\cdot r\cdot x$	from mole balance of oxygen
2	$\frac{\partial x}{\partial t}=\frac{\dot{V}_E.r}{U-u}\cdot x_w-\frac{p_0}{p}\cdot\frac{\dot{v}_0}{U-u}\cdot(1-x_w)-\frac{\dot{V}_E.r}{U-u}\cdot x$	ordinary first order differential equation
3	$\forall_{a=\frac{\dot{V}_E.r}{U-u}\cdot x_w-\frac{p_0}{p}\cdot\frac{\dot{v}}{U-u}(1-x_w)\Lambda\ b=\frac{\dot{V}_E.r}{U-u}}\quad dx=(a-b\cdot x).dt$	from 2°
4	$\frac{dx}{b\cdot x-a}+dt=0$	from 3°
5	$\int\frac{dx}{b\cdot x-a}+\int dt=const=c$	from 4° indefinite integral definition
6	$\frac{1}{b}\cdot ln\lvert b\cdot x-a\rvert+t=c$	from 5°
7	$ln\lvert b\cdot x-a\rvert\equiv b\cdot(c-t)=c'-b\cdot t$	c′−new constant of integration
8	$e^{(c'-b\cdot t)}=c''\cdot e^{-b\cdot t}\equiv b\cdot x-a$ C˝-new constant of integration	from 7° and natural logarithm definition

9	$\forall_{t\to\infty} \quad x \to x_0 \Rightarrow c^{"} = b\cdot x_0 - a \Rightarrow \left(b\cdot x_0 - a\right)\cdot e^{-b\cdot t} = b\cdot x - a$ $\exists_{\frac{a}{b}=x_w-\varepsilon_p\cdot\varepsilon\cdot\varepsilon_k} \quad x\left(t\right) = \left(x_0 - \frac{a}{b}\right)\cdot e^{-b\cdot t} + \frac{a}{b}$	from 8° i 3° $\varepsilon_k = \frac{1-x_w}{r}$ $\varepsilon_p = \frac{p_0}{p}$ $\varepsilon = \frac{\dot{\nu}}{\dot{V}_E}$
10	$x\left(t\right) = x_w - \varepsilon_p\cdot\varepsilon\cdot\varepsilon_k + \left(x_0 - x_w + \varepsilon_p\cdot\varepsilon\cdot\varepsilon_k\right)\cdot\exp\left(-\frac{\dot{v}_0.r^2}{\varepsilon_0\cdot u}\cdot t\right)$	from 2°, 8° i 9° q.e.d.

Note: a)Assumption, b)Thesis where: p -pressure at diving depth, p_0 -normal pressure, R -universal gas constant, T-absolute temperature, U-volume of the large bag, u-volume of the small bag, r-volume ratio of the small bag to the large bag $r = \frac{u}{U-u}$, $\dot{V}_E$ -ventilation of lungs, $\dot{\nu}_0$ -stream of consumed oxygen, x -molar fraction of oxygen inhaled breathing gas, x_w -molar fraction of oxygen in fresh breathing gas, x_0 -molar fraction of oxygen breathing space before diving apparatus is started, $\varepsilon_k = \frac{1-x_w}{r}$ -module representing structural solution of breathing space in diving apparatus, $\varepsilon = \frac{\dot{\nu}}{\dot{V}_E}$ -breathing module, $\varepsilon_p = \frac{p_0}{p}$ -pressure module, $\frac{\partial x}{\partial t}$ -elementary change in oxygen content ∂x in large bag with elementary time change ∂t.

Criterial Numbers

In Equation 1, the coefficient $\varepsilon_k = \frac{1-x_w}{r}$ is a formally dimensionless ratio constituting a criterion number describing the structural solution of the breathing space in the diving apparatus, whose value for a given solution is constant during the diving process $\varepsilon_k = idem \neq f\left(t\right)$, where time t refers to time of dive. It will be referred here as structural module ε_k. The ratio $\varepsilon_p = \frac{p_0}{p}$ is called here the pressure module which is the criterion number of external conditions. It decreases with diving depth $H: H \nearrow \Rightarrow \varepsilon_p \searrow$. With the increase of depth H, the pressure module ε_p has less and less effect on the decrease in the stable oxygen content in the breathing gas, which is breathed by the diver x_s, due to its inverse proportionality to pressure $p: x_s \sim \varepsilon_p = \frac{p_0}{p}$. For the constant depth H=idem the pressure module ε_p is constant value: $\varepsilon_p = idem \mid H = idem$. The module $\varepsilon = \frac{\dot{\upsilon}}{\dot{V}_E}$ is called here the breathing module, for standard pressure P_0 it will be marked as ε_0, connecting oxygen consumption $\dot{\nu}_0$ and ventilation of lungs $\dot{V}_E$ with work W_0 done by a man on surface $\dot{\upsilon}_0 = f\left(W_0\right)$. The use of the breathing module related to normal conditions gives an opportunity to develop a draft design of a diving apparatus, because these values are determined with good accuracy, but in general it should be assumed that the breathing module can be dependent on the pressure $\varepsilon = f\left(P\right)$. This assumption must, however, be verified experimentally, as the workload under water W consists of additional types of physical work which do not occur on the surface. For example, work done to overcome greater than in the air resistance associated with an increased pressure difference between the pulmonary centroid and the center of buoyancy of the breathing space in the apparatus in immersion. Expending extra effort to overcome the resistance associated with the higher density of breathing gas ρ, causing an increase in flow resistance $\Delta\rho$ posed by elements of the breathing space in the apparatus is also significant. In the breathing space of the diving apparatus there are also spatial obstructions that decrease the ventilation in some spaces, so-called dead spaces, whose impact on the safety of decompression should also be taken into account. Another condition that is difficult to determine unequivocally is an individual person-dependent phenomenon known as *bradycardia*. Associated with this slowed down breathing frequency occurring in immersion, reducing ventilation of lungs $\dot{V}_E$. Hence, the assumption that the breathing module ε should depend on pressure $\varepsilon = f\left(\varepsilon_p\right)$. It follows from the many years of experience that the breathing module ε_i is also an individualized function of morphology of a diver i .

Stabilization

An important factor is the speed of the system's response to the stepwise forced change in the partial pressure of oxygen $\rho(t)$ in the circulating breathing gas. For this purpose, the ventilation model (Equation 2) can be multiplied by pressure p present at diving depth $H: p(t) = p\cdot x(t)$. After ordering, the following can be written:

$$p(t) = p\cdot x_w - p_0\cdot\varepsilon_0\cdot\varepsilon_k + \left(p\cdot x_0 - p\cdot x_w + p_0\cdot\varepsilon\cdot\varepsilon_k\right)\cdot\exp\left(-\frac{\dot{\upsilon}_0\cdot r^2}{\varepsilon\cdot u}\cdot t\right)$$

(3)

where: $p(t)$ -partial pressure of oxygen in function of time t, p -pressure present at diving depth H

Using Equation 3 it is possible to estimate stabilization times $t_s(\Delta p_{kr})$ needed to achieve the stable value of oxygen partial pressure p_s in the breathing space of the diving apparatus with accuracy up to the specified critical pressure difference: $\Delta p_{kr} = p(t) - p(t\to\infty)$. The stabilization times calculated in this way are close to stabilization times $t_s(\Delta p_{kr})$ in SCR with a nozzle constant dosing system. The theoretical process of oxygen partial pressure stabilization p(t) in the breathing space of the SCR CRABE SCUBA is shown in Figure 3 and an example of real stabilization is shown in Figure 4.

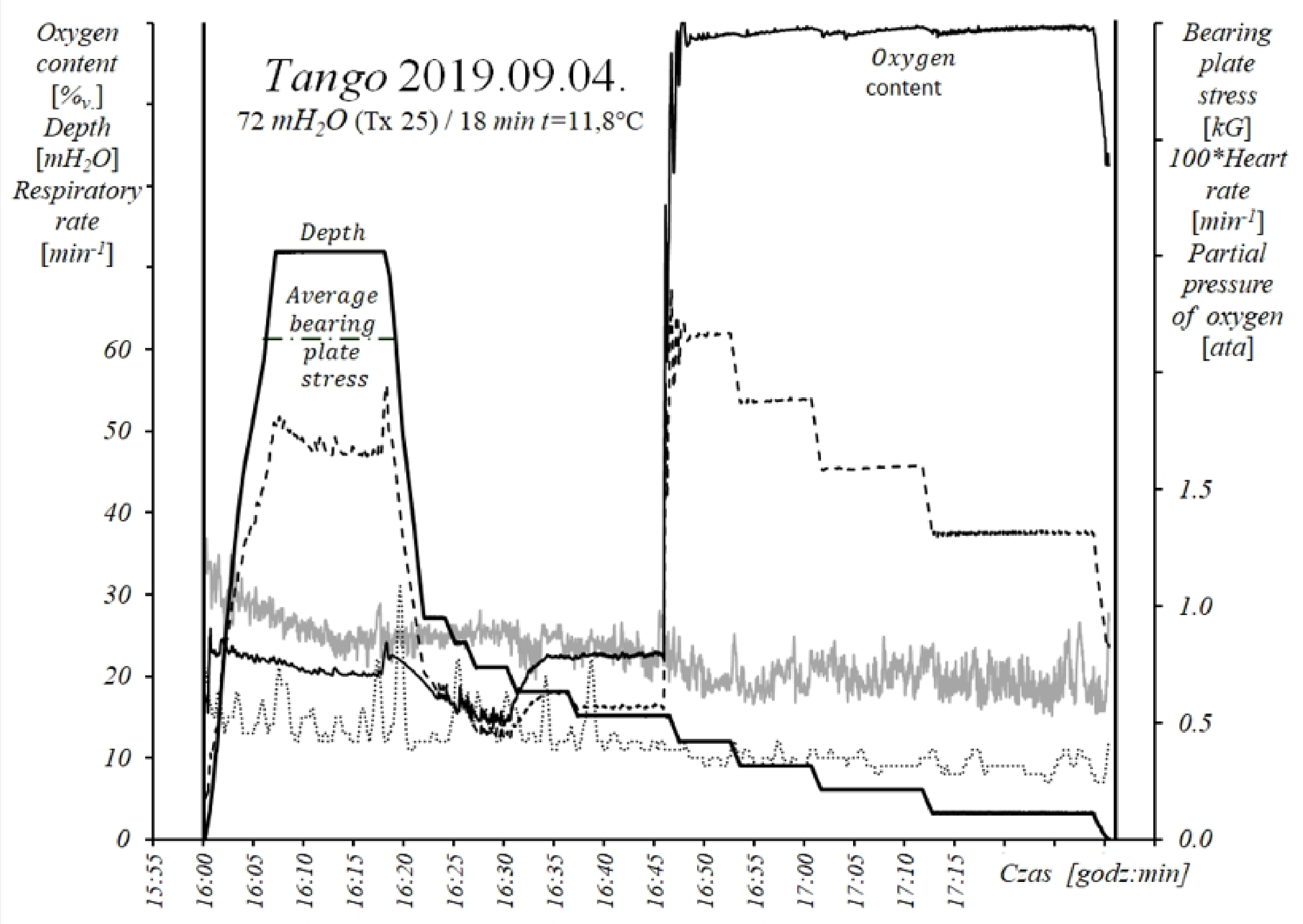


Figure 3: Example of the process of stabilizing the oxygen content $x(t)$ and its partial pressure $p(t)$ during a dive with SCR CRABE SCUBA apparatus.

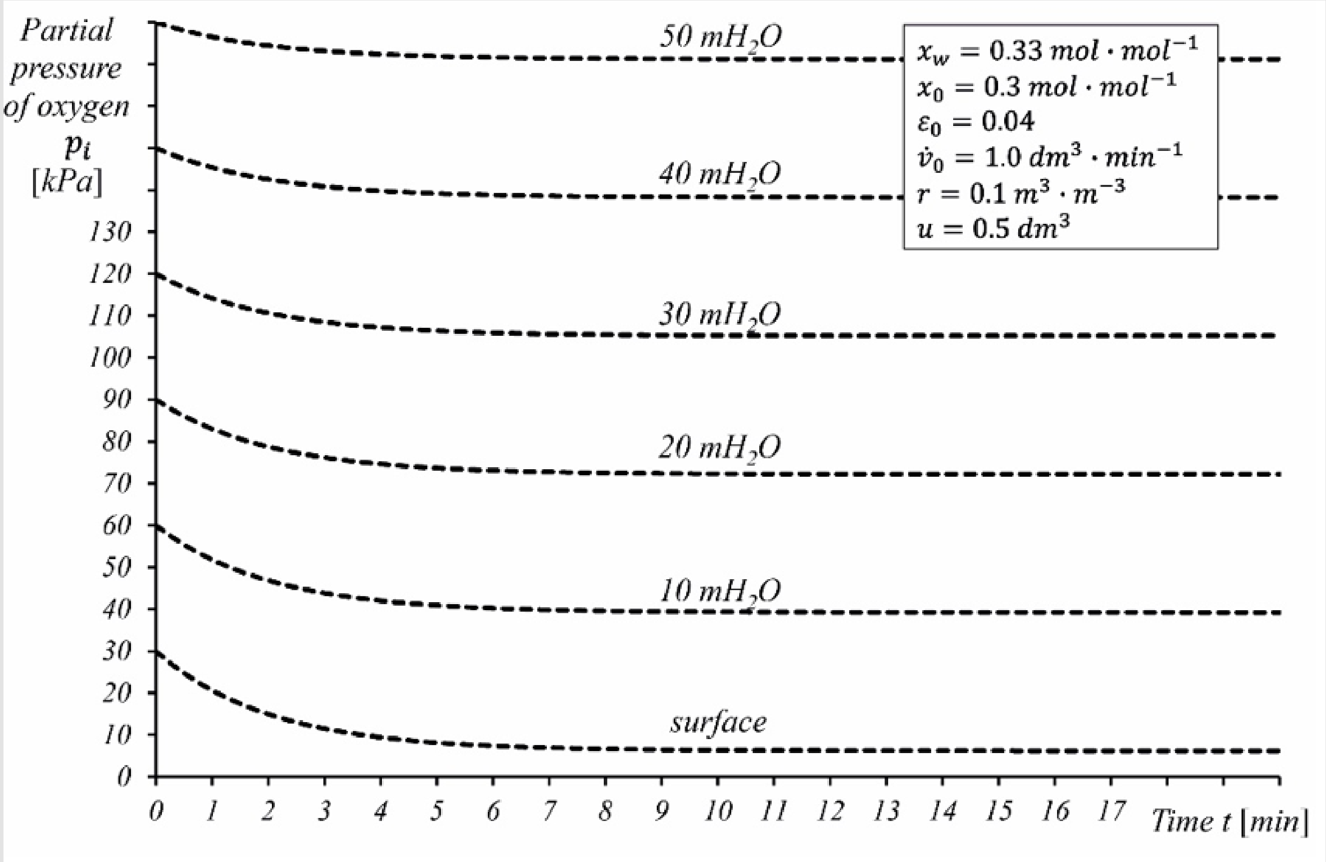


Figure 4: Theoretical times of partial pressure stabilization p(t) of oxygen in the large bag.

Stable Content

Equation $x(t \to \infty) = x_s = x_w - \varepsilon_p \cdot \varepsilon \cdot \varepsilon_k$ derived from an analysis of the model (2):

$$x_s = x_w - \frac{1-x_w}{r} \cdot \frac{\dot{v}}{\dot{V}_E} \cdot \frac{p_0}{p} = x_w - \varepsilon_p \cdot \varepsilon \cdot \varepsilon_k \,|\, t \to \infty \quad (4)$$

For the assumption ε=idem, dependence $x_s = f_{x_{w,\varepsilon,r}}(\varepsilon_p)$ is equation of straight-line $y = \alpha \cdot x + \beta$ with negative directional coefficient $\alpha = -\varepsilon_k \cdot \dot{\nu}$. Thus, for the domain of positive nmbers it is a monotonically decreasing function. The transformed Equation 4 leads to dependence $x_w - x_s = f_{x_{w,\varepsilon,r}}(p)$, which is the hyperbolic power function decreasing together with depth H, always greater than zero , because $\varepsilon_k \geq 0 \wedge \varepsilon \geq 0$.

Modeling

Transforming boundary mathematical model $t \to \infty$ (Equation 4) the following equation can be obtained: $\frac{p \cdot x_w - p_s}{p_0} = \frac{\Delta p}{p_0} = \frac{1-x_w}{r} \cdot \frac{\dot{\upsilon}}{\dot{V}_E}$. This equation can be further transformed into the form: $\frac{p \cdot x_w - p_s}{p_0} = \frac{\Delta p}{p_0} = \frac{1-x_w}{r} \cdot \frac{\dot{\upsilon}}{\dot{V}_E}$, where p represents the pressure at diving depth H, p_s is the actual partial pressure of oxygen in the breathing gas inhaled by the diver for determined structural conditions ε_k =idem and diving conditions $\varepsilon_p, \varepsilon = idem$, $\Delta p = p \cdot x_w - p_s$ is the difference in partial pressure of oxygen in the premix and the breathing gas inhaled by the diver. For different depths H, the breathing module $\varepsilon = \frac{\dot{\upsilon}}{\dot{V}}$ of oxygen consumption for lung ventilation $\dot{V}_E$ is function of diving depth $\varepsilon = f(H)$:

$$\forall_{\varepsilon_p, \varepsilon, \varepsilon_k = idem}\ p \cdot x_w - p_s = \Delta p = \varepsilon \cdot \frac{1-x_w}{r} \cdot p_0 \quad (5)$$

where: ε-the breathing module being the ratio of oxygen consumption to lung ventilation $\varepsilon = \frac{\dot{\upsilon}}{\dot{V}_E}$ at diving depth H

Interesting from the point of view of applications used for predicting safe decompression is a model combining total pressure p at diving depth H with oxygen partial pressure p_s which can be obtained by multiplying stable content x_s from dependence (Equation 4) by hydrostatic pressure p at diving depth H:

$$p_s = x_w \cdot p - \varepsilon_k \cdot \varepsilon \cdot p_0 \quad (6)$$

For $x_w, \varepsilon, r = idem$, dependence (Equation 6) is equation of straight-line $y = \alpha \cdot x + \beta$ having positive directional coefficient $\alpha = x_w$. Therefore, it should be a monotonically increasing function, with positive values for positive arguments, which is confirmed by the results of our own research, as shown in Figure 3.

Apparatus Design Module

The diving apparatus design module $\varepsilon_k = \frac{1 - x_w}{r}$ can be determined from direct measurement of the volume ratio of the small bag and the large bag $U : r = \frac{r}{U - u}$ and oxygen content in premix x_w. The volume of breathing bags in these apparatuses was made using a vial having volume $V = (500 \pm 5) cm^3$. As it was necessary to use the vial six times to measure the maximum volume, the minimum systematic error of the volume measurement was not lower than $\Delta V > 30 cm^3$. On the basis of the measurements, the volume of the small bag was assumed at the level of: $u = (0.50 \pm 0.01) dm^3$ and of the large sack $u = (5.30 \pm 0.03) dm^3$. Hence, the ratio of sack volumes can be assumed at the level $= (0.104 \pm 0.003) \frac{dm^3}{dm^3}$.

Breathing Module

Figure 3 presents an example of the experimental diving result for premix $x_w = 0.325 Nx$. The dives were used to determine the stable oxygen content x_s and total pressure p, constituting the basis for calculating stable value of oxygen partial pressure p_s. Using the known values of the structural module $\varepsilon_k = \frac{1 - x_w}{r}$, the pressure module $\varepsilon_p = \frac{p_0}{p}$, the mean value of the breathing module $\bar{\varepsilon} \cong 0.065$ and using dependence (6), the theoretical dependence can be drawn of the stable oxygen partial pressure p_s in the function of pressure p present at the maximum diving depth H. It should be noted, however, that the value of total pressure p recorded during the experiments was the pressure exerted by the atmosphere of the hyperbaric chamber, and the diving apparatus was additionally placed in the pool at the depth of approx. $h \cong 0.5 mH_2O$. A satisfactory compatibility can be achieved between the measurement results and the theoretical model of ventilation (Equation 6) when recorded total pressure p is increased by the value of $p_+ = 5 kPa$ associated with immersion of the apparatus below the surface of water in the pool - Figure 5. The results of measurements were used to estimate the volume ratio of the bags in the apparatus: $r = (0.104 \pm 0.003) m^3 \cdot m^{-3}$. The value of the structural module ε_k during tests for the oxygen content in the premix $x_w \cong 0.325 m^3 \cdot m^{-3}$ did not change $\varepsilon_k = idem$ was $\varepsilon_k = \frac{1 - x_w}{r} \cong \frac{1 - 0.325}{0.104} \cong 6.49$. The recorded stable values of oxygen partial pressures p_s for n=26 experimental dives from different years were showing high value of the determination coefficient $R^2 \cong 0.9785$, relatively rarely recorded in biological studies. It can be assumed that a good agreement between the theoretical and experimental results is obtained, despite the fact that the amount of work done by the divers was not constant $W \neq idem$, hence for the investigations the effect of the pressure applied to the underwater ergometer plate F on the stable oxygen content p_s did not seem significant - Figure 6.

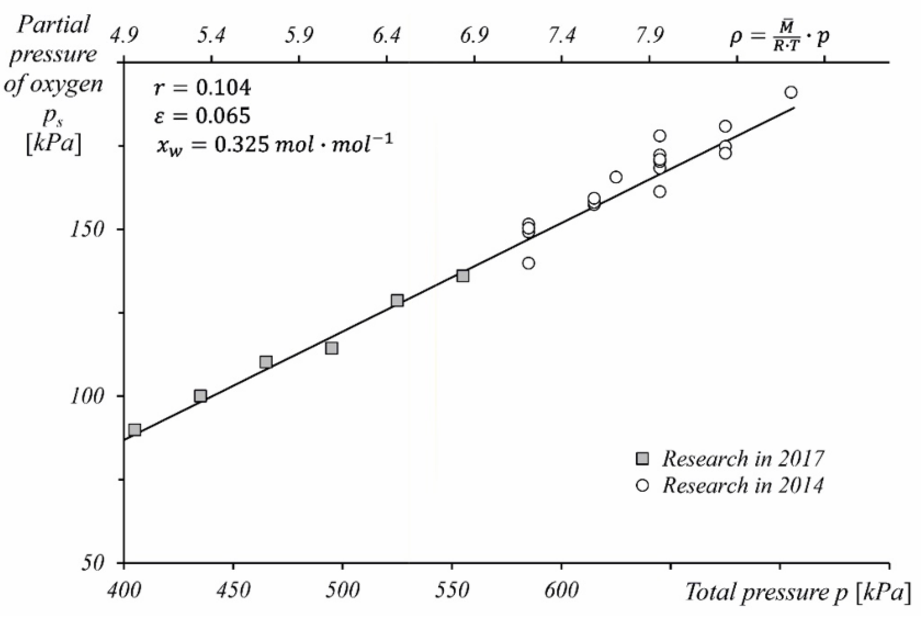


Figure 5: Practical values of stabilized partial pressure of oxygen p_s in the function of pressure p present at the depth H and density ρ of breathing gas $p_s = f(p)$, based on our own preliminary research carried out in 2014 and 2017; where: ρ-density of the breathing gas $p = f(p)$, $\overline{M}$ -mean molar mass of the breathing gas, R -universal gas constant, T -thermodynamic temperature.

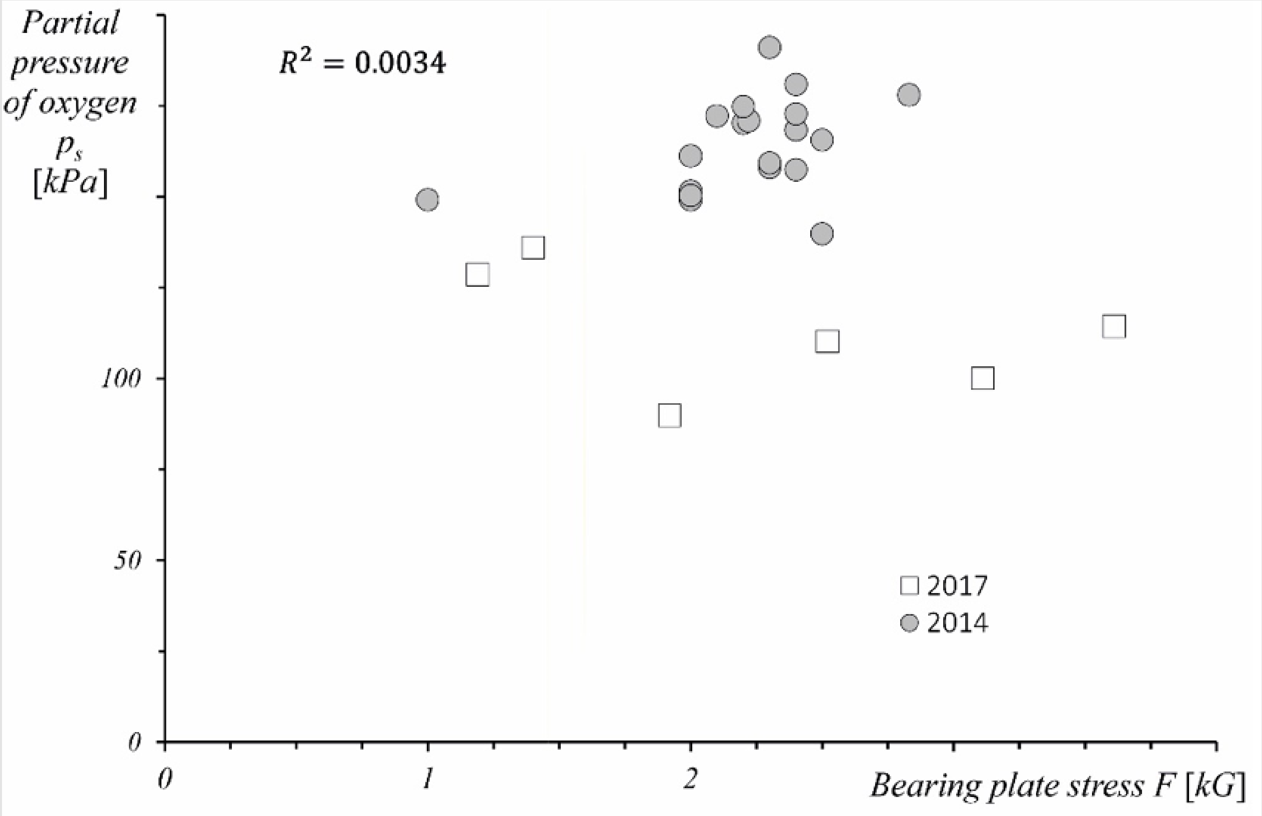


Figure 6: Results of measurements of the stress F applied to the underwater ergometer plate for the same dives as in Figure 5.

Simulator

The basis for modeling diving technologies is checking decompression distributions. For modeling decomposition distributions, it is important to check changes in the breathing module ε in function of depth: $\varepsilon = f(H)$. On this basis, the accuracy of ventilation model of the breathing space in the diving apparatus can be determined. The ventilation model can be used to determine stable oxygen content x_s for individual depths $H : x_s = f(H)$. When the fluctuation of stable oxygen content x_s is taken into account for individual depths H, this model contributes to planning adequate decompression together with the *central nervous syndrome* model, determining the risk of oxygen toxicity in planned exposures. By grouping the premises derived from the ventilation and *central nervous syndrome* models and using the decompression model, exposure distributions can be determined in the form of decompression tables that are verified through experiments with participation of human beings. Use of a breathing simulation stand together with metabolic gas exchange in hyperbaric conditions can reduce the risk of big errors in the initial phase of investigations, by providing answers to a number of important problems related to stability of composition of a breathing gas in the breathing space of a diving apparatus before experiments involving humans are commenced - Figure 7. Simulation of gas exchange in the process of breathing in hyperbaric conditions, in addition to mechanical simulation of the breathing cycle, should allow for intake of oxygen O_2 from the breathing space of the UBA and emission of carbon dioxide CO_2 and water vapor H_2O in the required ratio with the required accuracy and precision. A metabolic simulator coupled with a breathing simulation stand and a hyperbaric simulator can be used for the study and validation of mathematical models of the ventilation process in the underwater breathing apparatus. In simple metabolic simulators, controlled dosing of carbon dioxide CO_2 is used, disregarding the resultant increase in gas mass in the hyperbaric breathing space. Such a procedure is justified when mass stream $\dot{m}_{co_2}$ of carbon dioxide CO_2 is so small in relation to mass flow of the ventilation medium $\dot{m}$ and the whole mass in the ventilated breathing space $\dot{m}_o$ exchange, that the carbon dioxide CO_2 metering $\dot{m}_{co_2}$ does not disturb the ventilation process to a significant extent. Oxygen can be removed by catalytic oxidation of various chemical compounds added to the breathing space. Such oxygen intake may be accompanied by simultaneous emission of carbon dioxide CO_2 and water H_2O. This is currently one of the best methods for simulating gas exchange. As has been mentioned, the mapping of the breathing process in hyperbaric conditions is connected not only with the possibility to carry out investigations under increased pressure and mechanical simulation of lung ventilation, but also with mapping of gas exchange. These processes should be reproduced with reproducibility, accuracy and precision required by the scientific research method. However, they cannot simulate all the details of the real human-machine ergonomic interaction, offering only a simplified model, but adequate to the research project undertaken. In the case of simulation of gas exchange taking place in the body, the most important is oxygen consumption of $\dot{V}_{o_2}$ and the emission of carbon dioxide $\dot{V}_{co_2}$ in its place. The most common are the following two values of the adopted respiratory quotient $\varepsilon_{RQ} \in \{0.75; 0.8\} dm^3 \cdot dm^{-3}$ indicating the volume ratio $\varepsilon_{RQ} = \frac{\dot{v}_{co_2}}{\dot{v}_{o_2}} \varepsilon$ at which emission $\dot{V}_{co_2}$ of carbon dioxide CO_2 in place of consumed $\dot{V}_{o_2}$ of oxygen O_2 occurs. To simulate this process, catalytic oxidation of ethanal CH_3CHO or propanone $(CH_3)_2CO$ can be used:

$$\begin{aligned} &2 \cdot CH_3CHO + 5 \cdot O_2 \xrightarrow{catalyst} 4 \cdot CO_2 + 4 \cdot H_2O \qquad \varepsilon_{RQ} = 0.80 \\ &(CH_3)_2 CO + 4 \cdot O_2 \xrightarrow{catalyst} 3 \cdot CO_2 + 3 \cdot H_2O \qquad \varepsilon_{RQ} = 0.75 \end{aligned} \quad (7)$$

where: ε_{RQ} *-respiratory quotient*

Figure 7: Stand of combined simulators: respiratory, metabolic and hyperbaric.

The catalyst in reactions (Equation 7) can be platinum Pt on a ceramic carrier, diatomaceous earth, alumina, etc. The reaction takes place in the volume of the breathing gas drawn in by the breathing simulator in the reactor and then the gas is returned together with combustion products to the breathing space in the SCR placed in the hyperbaric chamber. In the reaction (Equation 7) the volume of liquid substrates is so small in relation to the volume of the gaseous products formed and the entire breathing space of the SCR that most often it does not cause any serious disturbances, significant for the balance of the volume of the breathing space. In addition, the gas is humidified, which makes the simulation process more real and creates good chemisorption conditions on the filling of the carbon dioxide CO_2 absorber used in the SCR. The significant exothermic effect of the oxidation process may generate some problems, the hence reaction products should be cooled once they leave the reactor, before the breathing gas returns to the breathing space of the SCR. Implementation-oriented studies on the currently used metabolic simulator was presented earlier (Kłos R [2]). The catalytic oxidation of propanone was selected for simulation of metabolic processes. Propanone $(CH_3)_2CO$ was metered to the reactor by means of SMART Digital S pump DDA made by Danish company Grundfos Holding A/S, which offers dosing with accuracy required for the counterpressure occurring during the investigations. The reactor was filled with $0.5\%_m Pt$ catalyst bed on alumina type 73 made by *Johnson Matthey from Great Britain* in the form of cylinders, having about $3.25mm$ in diameter and $3.48mm$ in height. The research was based on volumetric measurements, which for liquid such as propanone $(CH_3)_2CO$ are dependent on temperature t and for oxygen O_2 also from pressure p. Most often the measurements were made at temperature of approx. $t \cong 25^\circ C$. During the measurements constant pressure value $p = const$ was maintained. Oxygen consumption calculations were referenced to the temperature and pressure present at level of inhalation from the mouthpiece of the dive apparatus $\{t, p\}$, and referenced to standard conditions $T_0 \cong 273.15K; p_0 \cong 101.325kPa$. For each measurement, the molar volume of oxygen was determined depending on the temperature present in the respiratory circuit of the diving apparatus. Propanone $(CH_3)_2CO$ is characterized by molar mass $M_{Me_2CO} \cong 58.08kg\cdot kmol^{-1}$, density $d_{20^\circ C} \cong 0.791kg\cdot dm^{-3}$. It follows from reaction (Equation 7) that $1mol$ of Me_2CO reacts with 4 moles of oxygen CO_2. It follows from the above considerations that the breathing module ε will be:

$$\varepsilon \equiv \frac{\dot{v}_0}{\dot{V}_E} = 4\cdot\frac{\nu_0\cdot\dot{V}_{Me_2CO}\cdot d_{25^\circ C}}{M_{Me_2CO}\cdot V_E\cdot f\cdot\frac{p}{p_0}\cdot\frac{T_0}{T}} \quad (8)$$

where: v_0 -molar volume of oxygen under normal conditions, V_{Me_2CO} -volume flow of propanone, $\dot{V}_E$ -ventilation flow referred to standard conditions, $d_{25^\circ C}$ -density of propanone at temperature $t = 25^\circ C$, M_{Me_2CO} -molar mass of propanone, V_E -cylinder capacity of the piston breathing device, f-piston frequency of the respiratory device, p-pressure under measurement conditions, T-temperature under measurement conditions, $T_0 \cong 273.15K$ and $P_0 \cong 101.325kPa$

Results/Observations

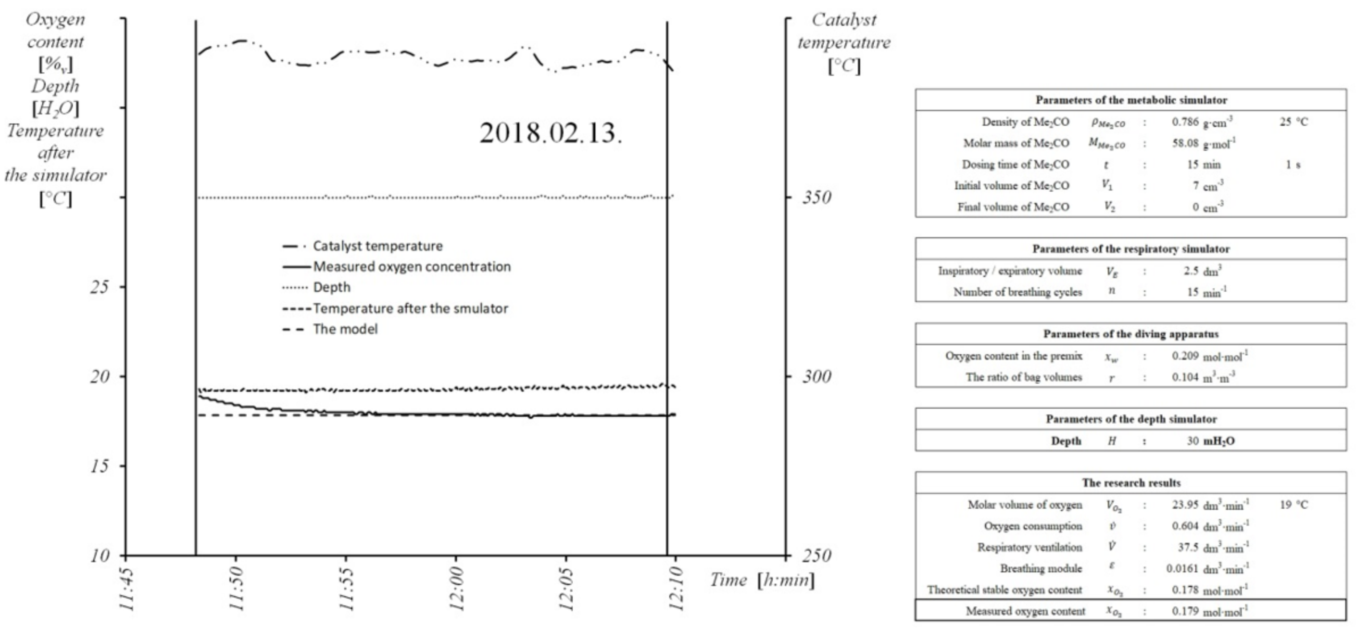
The experiments concerning the mathematical ventilation models were performed during manned and unmanned experimental diving aimed at verification of the mathematical model of ventilation. The experiments were carried out for semi-closed rebreather SCR CRABR SCUBA.

Unmanned Investigations

Air was used for supply the SCR CRABE SCUBA. The results of some simulation tests are shown in Table 2 together with the comparison of the theoretical values derived from the model (Equation 8). Figure 8 shows examples of the results from a single study.

Table 2: Example of the results of measurements of stable oxygen content x_s into breathing loop of SCR CRABE SCUBA supplied by air for various parameters of the metabolic simulator and respiratory settings (volume of inspiration $V_E = 1.5dm^3$; piston frequency $f = 15\min^{-1}$).

Depth	Temperature	Respiratory module	Parameters of metabolic simulator		Stable content of oxygen	
			Acetone volume	Time	Theoretical	Measured
$[mH_2O]$	$[^\circ C]$	$[m^3 \cdot m^{-3}]$	$[cm^3]$	$[\min : s]$	$[m^3 \cdot m^{-3}]$	
30	27.3	0.0702	21	17:14	0.072	0.072
40	28.3	0.0702	21.2	17:57	0.102	0.101
40	28.6	0.0703	21.2	17:57	0.102	0.101
50	28.5	0.0696	21	17:57	0.121	0.12
60	28.5	0.0682	21.4	18.40	0.135	0.137
30	29.4	0.0103	16.4	94:50	0.189	0.192
40	30	0.0105	10.2	58:10	0.193	0.195
50	31.5	0.0108	3.6	20:00	0.195	0.196
60	32.8	0.0117	5	25:50	0.196	0.198
40	31.1	0.0823	19.4	14:09	0.084	0.085
50	30	0.0826	20.4	14:46	0.104	0.102
50	29.6	0.0825	20.4	14:46	0.104	0.104
60	29.4	0.08	19.8	14:46	0.122	0.12



Parameters of the metabolic simulator				
Density of Me_2CO	ρ_{Me_2CO}	:	0.786 $g\cdot cm^{-3}$	25 °C
Molar mass of Me_2CO	M_{Me_2CO}	:	58.08 $g\cdot mol^{-1}$	
Dosing time of Me_2CO	t	:	15 min	1 s
Initial volume of Me_2CO	V_1	:	7 cm^{-3}	
Final volume of Me_2CO	V_2	:	0 cm^{-3}	

Parameters of the respiratory simulator			
Inspiratory / expiratory volume	V_E	:	2.5 dm^3
Number of breathing cycles	n	:	15 min^{-1}

Parameters of the diving apparatus			
Oxygen content in the premix	x_w	:	0.209 $mol\cdot mol^{-1}$
The ratio of bag volumes	r	:	0.104 $m^3\cdot m^{-3}$

Parameters of the depth simulator			
Depth	H	:	30 **mH_2O**

The research results				
Molar volume of oxygen	V_{O_2}	:	23.95 $dm^3\cdot min^{-1}$	19 °C
Oxygen consumption	$\dot{v}$	:	0.604 $dm^3\cdot min^{-1}$	
Respiratory ventilation	$\dot{V}$	:	37.5 $dm^3\cdot min^{-1}$	
Breathing module	ε	:	0.0161 $dm^3\cdot min^{-1}$	
Theoretical stable oxygen content	x_{O_2}	:	0.178 $mol\cdot mol^{-1}$	
Measured oxygen content	x_{O_2}	:	0.179 $mol\cdot mol^{-1}$	

Figure 8: Example of the results of investigations on ventilation in SCR CRABE SCUBA.

The Pressure Tests in Immersion

The pressure tests carried out in the swimming pool inside the land-based diving complex were the next stage of research carried out to verify the design assumptions - Figure 9. The divers swam with the average work rate. The divers tried to maintain work rate at the level that did not produce breathing disturbances, if this was not the case the experiment was to be interrupted. The subjects started swimming after pressurisation of the chamber. During the experiments, the divers were equipped with the SCR CRABE SCUBA diving apparatus. The diving apparatus was hose supplied with nitrogen-oxygen or nitrogen-helium-oxygen gas mixture containing differ contents of oxygen and helium. The experiments were carried out at different depths of diving $[0;\ 80]\,mH_2O$.

Figure 9: Experimental diving complex with swimming simulator.

Discussion

As shown by the research data presented in Table 2, it can be expected that the model (Equation 3) is a sufficiently accurate approximation of the ventilation process of the diving apparatus and can be used to infer the parameters needed to plan the subsequent decompression. In this way the mathematical model of the process of ventilation of a SCR CRABE SCUBA has been proposed and validate it a special simulator of gas exchange in the breathing process. Developing this model with the required accuracy provides the opportunity to work out data for the assumed decompression model. The proposed model has been adapted to the process of ventilation in hyperbaric chambers and generalized of the adopted method for the submarine ventilation process. The research on ventilation of a mining excavation constituted the validation of the adopted research approach with the success. These experimental work results will be presented in next articles in this series.

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