

Oscillatory Diffusion and Chemical Vortices in Multicomponent Plasma

Vadim Urpin*

A.F. Ioffe Institute of Physics and Technology, 194021 St. Petersburg, Russia

*Corresponding author: Vadim Urpin, A.F. Ioffe Institute of Physics and Technology, 194021 St. Petersburg, Russia



ARTICLE INFO

Received: 📅 September 13, 2021

Published: 📅 October 08, 2021

Citation: Vadim Urpin. Oscillatory Diffusion and Chemical Vortices in Multicomponent Plasma. Biomed J Sci & Tech Res 39(2)-2021. BJSTR. MS.ID.006283.

ABSTRACT

Diffusion caused by electric currents and the Hall effect can lead to a formation of chemical inhomogeneities in plasma. This mechanism can be responsible for a formation of chemical spots in laboratory and astrophysical plasmas. Such diffusion can be accompanied by a very special type of magnetohydrodynamic waves in which only the number density of impurity oscillates. These waves can be called compositional waves. They exist if the magnetic field is sufficiently strong and the magnetic pressure in plasma is much greater than the gas pressure. **Keywords:** multicomponent plasma, magnetic field, linear waves, chemically peculiar vortices.

Keywords: Multicomponent Plasma; Magnetic Field; Linear Waves; Chemically Peculiar Vortices

Introduction

Often laboratory and astrophysical plasmas are multicomponent, and diffusion of elements is important for some phenomena. For instance, diffusion processes can lead to chemical inhomogeneities and, as a result, to change the emission, heat transport, and conductivity [1-3]. In thermonuclear fusion, the source of impurities is usually the chamber walls, and diffusion determines the distribution impurities [4-6]. Even a small number of heavy ions influences the rate of radiative losses in plasma and changes its thermal properties. In astrophysical conditions, chemical inhomogeneities have been detected in the surface layers of many stars of various spectral types. It is widely accepted that these inhomogeneities are determined by diffusion processes but, however, the mechanisms resulting information of chemical spots is still uncertain. Diffusion in plasma can differ qualitatively from that in neutral gases because of the presence of charged particles - electrons. This particularly concerns hydrogen plasma [7]. In such plasma, the influence of electrons on diffusion of heavy ions is especially pronounced.

Chemical inhomogeneities can appear because of various reasons, for instance, it is often thought that chemical spots occur

due to the presence of magnetic fields. The magnetic field B can magnetize electrons and produce a non-uniform distribution of ions. Anisotropy of transport is characterized by the Hall parameter, $x_e = \omega_{be} \tau_e$, where $\omega_{be} = eB / m_e c$ is the gyrofrequency of electrons and τ_e is their relaxation time. In hydrogen plasma, $\tau_e = 3\sqrt{m_e} (k_b T)^{3/2} / 4\sqrt{2\pi} e^4 n \Lambda$ [8] where n and T are the number density and temperature of electrons, respectively, L is the Coulomb logarithm. The current-driven diffusion can lead to a formation of chemical inhomogeneities like other diffusion processes. Using a simple model, we show that the interaction of the current and field with impurities leads to their diffusion in the direction perpendicular to both. This type of diffusion can contribute to formation of chemical spots even in a relatively weak magnetic field with $x_e \ll 1$. We also show that such diffusion can be accompanied by a particular type of waves in which only the impurity number density oscillates.

Formulation of the Problem and Basic Equations

Consider a cylindrical magnetic configuration with the field in a z -direction, $\vec{B} = B(s)\vec{e}_z; (s, \varphi, z)$ and $(\vec{e}_s, \vec{e}_\varphi, \vec{e}_z)$ are cylindrical

coordinates and the corresponding unit vectors. Then, the electric current is given by

$$j_\phi = -(c/4\pi i)(dB/ds). \tag{1}$$

If $j_\phi \rightarrow 0$ at large s then $B \rightarrow B_0 = \text{const}$ at $s \rightarrow \infty$. Note that such magnetic configurations can be unstable for some dependences $B(s)$ [9-11] but we assume that the instability does not occur for chosen dependencies $B(s)$. Plasma is assumed to be fully ionized and consists of electrons e , protons p , and small amount of heavy ions i . A small admixture of heavy ions does not influence the dynamics of a background hydrogen plasma and these ions can be considered as test particles. The partial momentum equations in ionized plasma have been discussed by [7,12]. The paper [12] considers transport processes in the hydrogen-helium plasma but the derived equations describe the hydrogen plasma with a small admixture of heavy test ions. Then, the partial momentum equation for ions i has the form

$$m_i n_i \vec{v}_i + m_i n_i \vec{v}_i \cdot \nabla \vec{v}_i = -\nabla p_i + Z_i e n_i \left(\vec{E} + \frac{\vec{V}_i}{c} \times \vec{B} \right) + \vec{R}_{ie} + \vec{R}_{ip} + \vec{F}_i, \tag{2}$$

where \vec{v}_i is the velocity, Z_i is the charge of the species i , p_i and n_i are the partial pressure and number density, respectively, and \vec{E} is the electric field; \vec{F}_i is the external force on species i . We neglect the external force in our model since its influence is often insignificant. The forces \vec{R}_{ie} and \vec{R}_{ip} are caused by the interaction of ions i with electrons and protons, respectively. The forces \vec{R}_{ie} and \vec{R}_{ip} are internal and their sum over all components is zero.

The velocity \vec{v}_i can be represented as a sum of \vec{V}_i and \vec{V} where \vec{V} is the hydrodynamic velocity of plasma and $\sim V$ is a diffusive velocity of ions i relative plasma. Since in our model n_i is small compared to n_p , the hydrodynamic velocity is approximately equal to the mean velocity of protons, $\vec{V} \approx \vec{v}_p$. In this paper, we assume that background plasma is in hydrostatic equilibrium. Then, $\vec{V} \approx \vec{v}_p = 0$ and, hence, $\vec{v}_i \approx \vec{V}_i$. If the temperature is uniform, the friction force \vec{R}_{ip} has only a component proportional to the relative velocity of ions i and protons, $\vec{R}_{ip} \propto (\vec{v}_p - \vec{V}_i)$. This force can be easily calculated if $A_i = m_i / m_p \gg 1$. Since the velocity of a background plasma is zero, \vec{R}_{ip} can be represented as

$$\vec{R}_{ip} = (0.42 m_i n_i Z_i^2 / \tau_i) (-\vec{V}_i) \tag{3}$$

[12], where $\tau_i = 3\sqrt{m_i} (k_B T)^{3/2} / 4\sqrt{2\pi} 4n\Lambda$; τ_i / Z_i^2 is the characteristic time of scattering between protons and heavy ions; L is assumed to be the same for all types of scattering.

The friction force on the l.h.s of Eq. (2), \vec{R}_{ip} , can be estimated as $m_i n_i Z_i^2 V_i / \tau_i$ and the first term in this equation is $\sim m_i n_i V_i / \Delta t$ where Δt is the timescale of diffusion processes. Therefore, this term is much smaller than the friction force \vec{R}_{ip} if $\Delta t \gg \tau_i / Z_i^2$. The

second term on the l.h.s. of Eq. (2) is of the order of $m_i n_i V_i^2 / L$ where L is the characteristic length scale. Since $L \sim V_i \Delta t$, this term can also be neglected if $\Delta t \gg \tau_i / Z_i^2$. Then, Eq. (2) transforms into

$$-\nabla p_i + Z_i e n_i \left(\vec{E} + \frac{\vec{V}_i}{c} \times \vec{B} \right) + \vec{R}_{ie} + \vec{R}_{ip} + \vec{F}_i = 0 \tag{4}$$

If $n_i \ll n_p$, then, \vec{R}_{ie} is given by $\vec{R}_{ie} = -(Z_i^2 n_i / n) \vec{R}_e$ [12], where \vec{R}_e is the force acting on the electrons. Since $n_i \ll n$, \vec{R}_e is determined by scattering of electrons on protons but scattering of electrons on heavy ions gives a small contribution. Therefore, one can use for \vec{R}_e the expression obtained by [7] for one-component plasma. In the case of a cylindrical isothermal configuration, this force reads

$$\vec{R}_e = -\alpha_\perp \vec{v} + \alpha_\parallel \vec{b} \times \vec{u} \tag{5}$$

where $\vec{u} = -\vec{j} / en$ is the velocity of electric current; $\vec{b} = \vec{B} / B$; the coefficients α_\perp and α_\parallel have been calculated by Braginskii [7]. The force (5) is caused a friction due to a relative motion of the electron and proton gases. Then, Eq.(1) yields for the current velocity $\vec{u} = (c / 4\pi en)(dB/ds) \vec{e}_\phi$.

Consider the current-driven diffusion in a magnetic field with $x_e \ll 1$. Substituting \vec{u} into Eq.(5) and using coefficients $\alpha_{\perp,\parallel}$ we obtain with the accuracy in linear terms in x_e

$$R_{ie\phi} = Z_i^2 n_i \left(0.51 \frac{m_e}{\tau_e} u \right), R_{ies} = Z_i^2 n_i \left(0.21 x \frac{m_e}{\tau_e} u \right). \tag{6}$$

The momentum equation for the species i (Eq.(4)) depends on E_s and E_ϕ components of the electric field. The momentum equations for e and p , containing these electric fields, read

$$-\nabla(nk_B T) - en \left(\vec{E} + \frac{\vec{u}}{c} \times \vec{B} \right) + \vec{R}_e + \vec{F}_e = 0, \tag{7}$$

$$-\nabla(nk_B T) + en \vec{E} - \vec{R}_e + \vec{F}_p = 0, \tag{8}$$

Using Eq. (5), we obtain

$$E_s = -\frac{uB}{2c} - \frac{1}{e} \left(0.21 \frac{m_e u}{\tau_e} x \right), E_\phi = \frac{1}{e} \left(0.51 \frac{m_e u}{\tau_e} \right). \tag{9}$$

Substituting Eqs.(6), (3), and (9) into the s - and ϕ -components of Eq.(4), we arrive to the expression for a diffusion velocity, \vec{V}_i ,

$$\vec{V}_i = V_{is} \vec{e}_s + V_{i\phi} \vec{e}_\phi, V_{is} = V_{ni} + V_B, \tag{10}$$

$$V_{ni} = -D \frac{d \ln n_i}{ds}, V_B = D_B \frac{d \ln B}{ds}, V_{i\phi} = D_{B\phi} \frac{dB}{ds}, \tag{11}$$

V_{ni} is the standard diffusion velocity and V_B is the velocity of diffusion caused by the current. The corresponding diffusion coefficients are

$$D = \frac{2.4c_A^2\tau_i}{Z_i^2}, D_B = \frac{2.4c_A^2\tau_i}{Z_i A_i} (0.21Z_i - 0.71), \tag{12}$$

$$D_{B\phi} = 1.22 \sqrt{\frac{m_e c}{m_i}} \frac{(Z_i - 1)}{4\pi e n Z_i} \tag{13}$$

where $c_i^2 = k_B T / m_i$ and $c_A^2 = B^2 / (4\pi n m_p)$.

Distribution of Ions Under the Influence of Electric Currents

The condition of hydrostatic equilibrium reads in our model

$$-\nabla p + \vec{j} \times \vec{B} / c = 0, \tag{14}$$

where p and r are the total pressure and density of plasma, respectively. In hydrogen plasma, we have $p \approx 2nk_B T$ where k_B is the Boltzmann constant. Integrating Eq. (14) and assuming the temperature to be constant, we obtain

$$n = n_0 (1 + \beta_0^{-1} - \beta^{-1}), \tag{15}$$

where $\beta = 8\pi p_0 / B^2$; (p_0, n_0, T_0, β_0) are the values of (p, n, T, b) at $s \rightarrow \infty$.

Consider the distribution of elements in equilibrium. Since $V_{is} = 0$ in equilibrium, Eq.(10) yields

$$\frac{d \ln n_i}{ds} = \frac{D_B}{D} \frac{d \ln B}{ds} \tag{16}$$

The r.h.s. is responsible for the influence of currents on the distribution of impurities. One has from Eq. (16)

$$\frac{d}{ds} (nk_B T) = -\frac{B}{8\pi} \frac{dB}{ds} \tag{17}$$

Taking into account Eq. (16) and integrating Eq.(17), we have

$$\frac{n}{n_{i0}} = \left(\frac{n}{n_0} \right)^\mu, \tag{18}$$

where $\mu = -2Z_i(0.21Z_i - 0.71)$ and n_{i0} is n_i at $s \rightarrow \infty$. If the local abundance of i is $\gamma_i = n_i / n$, then taking into account Eq. (17), obtain

$$\frac{\gamma_i}{\gamma_{i0}} = \left(\frac{n}{n_0} \right)^{\mu-1} = \left(1 + \frac{1}{\beta_0} - \frac{1}{\beta} \right)^{\mu-1} \tag{19}$$

where $\gamma_{i0} = n_{i0} / n_0$. Local abundances depend essentially on the field strength. These dependences are particular sensitive to B in the case of heavy ions with large charge numbers. If other diffusion mechanisms are negligible, then the exponent $(\mu - 1)$ reaches large negative values for large Z_i and, as a result, strong abundance anomalies can be produced in this case. For instance, $(\mu - 1)$ is equal 1.16, -0.52, and -2.04 for $Z_i = 2, 3,$ and $4,$ respectively. Note that $(\mu - 1)$ changes its sign with an increase of Z_i : $(\mu - 1) > 0$ if $Z_i = 2$ but $(\mu - 1) < 0$ for $Z_i \geq 3$. Therefore, elements with $Z_i \geq 3$ are in deficit ($\gamma_i < \gamma_{i0}$) in the region with a weak magnetic field ($B < B_0$) but these elements should be overabundant in the region with a stronger field, $B > B_0$.

Eq. (20) describes the distribution of impurities in diffusive equilibrium. The characteristic timescale to reach this equilibrium, t_B , can be estimated as

$$t_B \sim L / V_B \sim L^2 / D_B, \tag{20}$$

where L is the length scale of the magnetic field, $L = |d \ln B / ds|^{-1}$. The characteristic timescale of baro-diffusion is given by the well-known expression

$$t_n \sim L / V_n \sim L^2 / D. \tag{21}$$

Hence, the current-driven diffusion operates on a shorter timescale if $D_B > D$ or

$$\frac{c_A^2}{c_s^2} > Z_i^{-1} (0.21Z_i - 0.71)^{-1}, \tag{22}$$

where c_s is the sound speed, $c_s^2 = k_B T / m_p$. Therefore, the current-driven diffusion is more efficient if the magnetic pressure exceeds the gas pressure.

Chemical Vortices

In our simplified model of cylindrical magnetic configuration and velocity (10), the continuity equation for ions i has the form

$$\frac{\partial n_i}{\partial t} - \frac{1}{s} \frac{\partial}{\partial s} (sD \frac{\partial \delta n_i}{\partial s} - s n_i \frac{D_B}{B} \frac{dB}{ds}) + \frac{1}{s} \frac{\partial}{\partial \phi} (n_i D_{B\phi} \frac{dB}{ds}) = 0. \tag{23}$$

Consider the behaviour of small perturbations of n_i by making use of a linear analysis of Eq. (23). In the unperturbed state, we assume that plasma is in a diffusive equilibrium and, therefore,, the unperturbed impurity number density satisfies Eq. (18). For the sake of simplicity, we assume that small perturbations are independent of z. Denoting disturbances of the impurity number density by δn_i and linearizing Eq.(23), we obtain the equation that describes the evolution of such small perturbations,

$$\frac{\partial \delta n_i}{\partial t} - \frac{1}{s} \frac{\partial}{\partial s} (sD \frac{\partial \delta n_i}{\partial s} - s \delta n_i \frac{D_B}{B} \frac{dB}{ds}) + \frac{1}{s} \frac{\partial}{\partial \phi} (\delta n_i D_{B\phi} \frac{dB}{ds}) = 0. \tag{24}$$

Consider only disturbances with a short radial wavelength. If this wavelength is shorter than the length scale of unperturbed quantities, then we can use the so-called local approximation. Then small disturbances are $\alpha \exp(-iks)$ where k is the radial wavevector, $ks \gg 1$. Since the basic state does not depend on time and ϕ , perturbations can be represented in the form $\delta n_i \alpha e^{i\omega t - iks - im\phi}$ where m is the azimuthal wavenumber and ω should be calculated from the dispersion equation. Such disturbances have a shape of cylindrical vortices of a chemical composition. Substituting δn_i into Eq. (24), we derive the dispersion equation for such perturbations in the form

$$i\omega = -\omega_k + i(\omega_l + \omega_n), \omega_k = Dk^2, \omega_l = kD_B (d \ln B / ds), \omega_n = (m/s) D_{B\phi} (d \ln B / ds). \tag{25}$$

This dispersion equation describes a special type of magnetohydrodynamic waves where only the number density

of impurities oscillates. Note that there are many different sorts of waves in magnetized plasma that can produce oscillations of a chemical composition. Such oscillations, for example, can be generated by standard magnetohydrodynamic waves [12] but, usually, fluctuations of impurities in such waves are accompanied by fluctuations of other hydrodynamic quantities. Indeed, the electric field has actively to take part in such waves and interact more efficiently with heavily charged ions than with protons. As a result, MHD waves can produce fluctuations of a chemical composition. However, these fluctuations are typically very small because the period of MHD waves is much shorter than the diffusion timescale. Chemical fluctuations become significant only if the period of a wave is comparable to the diffusion timescale.

The waves considered in this paper are caused by diffusion processes and their period is generally comparable to the diffusion timescale. Therefore, fluctuations of a chemical composition can reach a significant value in such waves. The quantity ω_R is responsible for decay of these waves with the timescale $\sim (Dk^2)^{-1}$ typical for standard diffusion. The frequencies ω_I and ω_H describe oscillations of the impurities caused by the combined action of electric current and the Hall effect. The considered waves are oscillatory if $|\omega_I + \omega_H| > \omega_R$. The latter condition is equivalent to

$$c_A^2 / c_s^2 > Z_i^{-1} |0.21Z_i - 0.71|^{-1} kL. \quad (26)$$

Therefore, the waves are oscillatory if the magnetic pressure is substantially greater than the gas pressure. The frequency of diffusion waves is higher in the region where the magnetic field has strong gradients. The order of estimate of ω_I is

$$\omega_I \sim kc_A (1 / Z_i A_i) (c_A / c_i) (l_i / L), \quad (27)$$

Where $l_i = c_i \tau_i$. Note that different impurities in such waves oscillate with different frequencies.

Conclusion

Diffusion of heavy ions in plasma under the influence of the current-driven mechanism has a number of features. Generally, the diffusion velocity can be comparable to (or even greater than) that caused by other diffusion mechanisms. The current-driven mechanism can lead to a formation of chemical spot even if the magnetic field is relatively weak. Note that other diffusion mechanisms usually require a much stronger magnetic field. The current-driven diffusion is caused by the Hall effect and electric current and, therefore, it leads to diffusion of heavy ions in the direction perpendicular to both the magnetic field and current. Therefore, a distribution of elements is determined by a geometry of the magnetic fields and electric currents. Chemical inhomogeneities can manifest themselves, for instance, by a non-uniform distribution of plasma temperature and emission. Note that this type of diffusion can be important also in some conductive

fluids if the magnetic field is sufficiently strong. Our consideration shows that a special type of magnetohydrodynamic waves may exist in multicomponent plasma.

These waves can be periodic if the magnetic field is sufficiently strong, and they are characterized by oscillations of the impurity number density alone. The frequency of such waves is rather low and is determined by a characteristic diffusion time. The order of magnitude estimates of this frequency yields $\omega_I \sim kD_B / L \sim c_A^2 \tau_i / A_i L \lambda$ where $\lambda = 2\pi / k$ is the wavelength. In stellar conditions, such waves can manifest themselves in peculiar magnetic stars where the magnetic field is $\sim 10^4$ G and the number density and temperature are $\sim 10^{14}$ cm⁻³ and $\sim 10^4$ K, respectively. If the length scale, L , and λ are of the same order of magnitude (for instance, $\sim 10^{11}$ cm), then the period of such compositional waves is $\sim 3 \times 10^3$ yrs. This is much shorter than the magnetic timescale and generation of such waves should lead to short-term spectral variability. Such waves can exist in laboratory plasmas as well, but their frequency is much higher in this case. If $B \sim 10^5$ G, $n \sim 10^{15}$ cm⁻³, $T \sim 10^6$ K, and $L \sim 1 \sim 10^2$ cm, then the period of compositional waves is $\sim 10^{-8}$ s. Note that frequencies of waves with various impurities can differ essentially since they depend on the sort of heavy ions.

In terrestrial conditions, the compositional waves also can manifest themselves by oscillations in spectra. In astrophysical conditions, the magnetic field has usually more complex topology than that considered in our model. In some cases, however, cylindrical magnetic configurations can mimic the magnetic field in certain regions. For example, the field near the magnetic poles in stars has approximately cylindrical symmetry [13] and our results describe qualitatively diffusion in the region near the poles. Therefore, the current-driven mechanism in combination with other diffusion processes contributes to a formation of chemical spots in various types of stars. For example, this mechanism can contribute to formation of element spots in Ap-stars where the magnetic fields have been detected. Also, many neutron stars have strong magnetic fields and topology of these fields is rather complex with spot-like structures at the surface. Such magnetic configurations can be responsible for formation of a spot-like chemical distribution at the surface. For example, evolution of neutron stars is very complicated, particularly, in binary systems [14] and, therefore, their surface chemistry can be complicated as well. Diffusion processes may play an important role in this chemistry.

Conflicts of Interest

The author declare no conflict of interest.

References

1. Vekshtein G (1987) Thermomagnetic effects in rarefied plasma. In Review of Plasma Physics: Ed. B.Kadomtzev; NY:120 Consultants Bureau, USA 15: 1-76.

2. Ren Y, Yamada M, Ji H, Gerhardt S, Kulsrud R (2008) Identification of the Electron-Diffusion Region during Magnetic Reconnection in a Laboratory Plasma. *Phys Rev L* 101: 5003-5006;
3. Kagan G, Tang XZ (2012) Electro-diffusion in a plasma with two ion species. *Phys Plasma* 19: 2709.
4. Losseva T, Popel S, Yu MY, Ma JX (2008) Ambipolar diffusion in complex plasma. *Phys Rev* 75(127): 6403-6407.
5. Molvig K, Vold E, Dodd E, Wilks S (2014) Nonlinear Structure of the Diffusing Gas-Metal Interface in a Thermonuclear Plasma. *Phys Rev L* 113: 9904-9907.
6. Ott T, Bonitz M (2011) Diffusion in a Strongly Coupled Magnetized Plasma. *Phys Rev L* 107: 50030-50030.
7. Braginskii S (1965) Transport Processes in a Plasma. *Reviews of Plasma Physics* 1: 205-363.
8. Spitzer L (1978) *Physical Processes in the Interstellar Medium*, Wiley: New York.
9. Tayler RJ (1973) The adiabatic stability of stars containing magnetic fields-I. Toroidal fields. *Mon Not R Astr Soc* 61: 365-377.
10. Bonanno A, Urpin V (2011) Resonance instability of axially symmetric magnetostatic equilibria. *Phys Rev E* 84: 6310.
11. Bonanno A, Urpin V (2011) The magnetohydrodynamic instability of current-carrying jets. *Astron Astrophys* 525A: 100.
12. Urpin V (1981) Transport phenomena and abundance anomalies in cosmic plasma. *Astrophys Sp Sci* 79: 11.
13. Urpin V, Chanmugam G, Sang Y (1994) Long-term evolution of crustal neutron star magnetic fields. *Astrophys J* 433: 780-785.
14. Urpin V, Konenkov D, Geppert U (1998) Evolution of neutron stars in high-mass X-ray binaries. *Mon Not R Astr Soc* 299: 73-77.

ISSN: 2574-1241

DOI: 10.26717/BJSTR.2021.39.006283

Vadim Urpin. Biomed J Sci & Tech Res



This work is licensed under Creative Commons Attribution 4.0 License

Submission Link: <https://biomedres.us/submit-manuscript.php>



Assets of Publishing with us

- Global archiving of articles
- Immediate, unrestricted online access
- Rigorous Peer Review Process
- Authors Retain Copyrights
- Unique DOI for all articles

<https://biomedres.us/>