

Modeling Parameter Selection Using A Random Generation Classification of Order Ten (10): Matlab Algorithm

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ABSTRACT

Model parameter selection in the process of mathematical modeling is a formidable problem which requires sophisticated algorithm indexed by a Matlab random generation function having a dominant 0.01 intensity with very few instances of higher random intensity for obvious reasons. This study have utilized the Matlab algorithm with a random function of order ten (10) having the structure: $v = m+0.01$ times $\text{rand}(10)$ with this random generation function. The novel result obtained shows boundary values of each parameter estimate, and have proven clearly why we have chosen our present model parameter values for the case of Port Harcourt HIV infection public health data that defines the number of target cells, number of infected cells, and number of viral load of the virions for every 30 day simplification for the purpose of HIV intervention strategy.

Keywords: Mathematical Modeling; Parameter Selection; Random Generation function; Matlab Algorithm; Numerical simulation

Introduction

Parameter estimation problem being an integral aspect of mathematical modeling has been a long standing problem [1-3]. However, there is urgent need to specify one of the alternatives of determining the estimated parameter values with particular reference with the modeling of HIV infection of CD4+ T- cells. In fast developing country like Nigeria where public health data are in acute shortage for purpose of designing a database that can be useful in the intervention of HIV infection, it is worthwhile to implement and apply this present numerical approach for the purpose of a numerical characterization of a time dependent viral load of the virions with the insight of forming HIV infection intervention database for the city of Port Harcourt in Nigeria. The factors that define the depletion of viral load of the virions over time and those that increase the viral load over time can provide useful insight for the intervention of this public health concerns. Some of the challenging and salient businesses of modelers in mathematical immunology are to develop models, methods of analysis and control of the immune system processes [4-6]. Starting with the classic research of Nwagor P et al. [7-9], on immune system

modeling and HIV-infection; this research in same vein also intends to examine further certain other properties of the already solved dynamic model which includes parameter estimation representing the HIV-infectio P n.

Time Dependent ODE Model

Following Nwagor (2020). A system of time dependent non-linear first ordinary differential equations is considered. In the absence of the delay term with $\tau=0$ then the model reduces to

$$\frac{dT}{dt} = S - dT + \alpha T \left(1 - \frac{T}{T_{\max}} \right) - \beta TV \quad (1)$$

$$\frac{dI}{dt} = \beta TV - \delta I \quad (2)$$

$$\frac{dV}{dt} = \rho I - cV \quad (3)$$

Where the notations T represents the number of target cells, I represent the number of infected cells, V represents the viral load of the virions at time t in units of days, s represents the rate

at which the new T cells are created from the source within the body such as the thymus, α represents the maximum proliferation rate of target cells. T_{\max} represents the T population density at which proliferation shuts off, d represents the death rate of the T cells. β represents the infection rate, δ represents the death rate of infective cells, ρ represents the rate of cure, c is the clearance rate constant of virions. The positive integer τ represents length of time measured in days. The system (1)-(3) satisfies the initial condition $T(0) \geq 0, I(0) \geq 0, V(0) \geq 0$

The possible non negative equilibrium of the system are $E_1(T^o, 0, 0)$ and $E_2(T^*, I^*, V^*)$

$$\text{Where } T^o = \frac{T_{\max}}{2\alpha} \left[(\alpha - d) \pm \sqrt{(\alpha - d)^2 + \frac{4\alpha s}{T_{\max}}} \right] T^* = \frac{\partial c}{\beta \rho}$$

$$I^* = \frac{1}{\beta \rho} \left[\beta \rho - cd + \alpha c - \alpha \delta c^2 / T_{\max} \right]$$

$$V^* = \frac{1}{c\beta^2 \delta T_{\max}} \left[\beta \rho - cd + \alpha c - \alpha \delta c^2 / T_{\max} \right]$$

$$\text{And } R_0 = \frac{T^o}{T^*}$$

Where R_0 defines the basic reproductive ratio, which determines the dynamical property of the system (1 - 3) over a long period of time.

MATLAB Numerical Scheme

The theory of MATLAB numerical scheme considered in this study involves the two-point boundary value problem of the equations of the form.

$$\frac{dy}{dx} = f(x, y)$$

Subject to the initial condition

$$\frac{dy}{dx} = y(x_o) = y_o$$

by increasing each step-in x by h , so that the n^{th} step becomes

$$x_n = x_o + nh$$

The Euler algorithm for the determination of the approximation

$$y_{n+1} \text{ to } y(x_{n+1}) \text{ is } y_{n+1} = y_n + f(x_n, y_n)h$$

The modified Euler method involves taking the gradient of the tangent line approximation at (x_n, y_n) the average of the gradient

$$(x_n, y_n) \text{ and } (x_{n+1}, y_{n+1})$$

so that the algorithm for the modified Euler for the determination of approximation is given by

$$y_{n+1} = y_n + \frac{h}{2} [f(x_n, y_n) + f(x_n + h, y_n) + f(x_n, y_n)h]$$

Also, the Runge-Kutta method which is based on a Taylor series approximation to the function $f(x,y)$ defines its algorithm for the determination of the approximation

$$y_{n+1} \text{ to } y(x_{n+1}) \text{ as}$$

$$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4),$$

Where

$$k_1 = hf(x_n, y_n)$$

$$k_2 = hf(x_n + \frac{1}{2}h, y_n + \frac{1}{2}k_1)$$

$$k_3 = hf(x_n + \frac{1}{2}h, y_n + \frac{1}{2}k_2)$$

$$k_4 = hf(x_n + h, y_n + k_3)$$

The Runge-Kutta method also extends to the solution of the problem of the type

$$\frac{dy}{dx} = f(x, y, z)$$

And

$$\frac{dz}{dx} = g(x, y, z)$$

Subject to the initial conditions

$$y(x_o) = y_o \quad z(x_o) = z_o$$

For a step of length h , and the n^{th} integration step, the Runge-Kutta algorithm for systems is of the form

$$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

And

$$z_{n+1} = z_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

Where

$$k_1 = hf(x_n, y_n, z_n)$$

$$k_2 = hf(x_n + \frac{1}{2}h, y_n + \frac{1}{2}k_1, z_n + \frac{1}{2}k_1)$$

$$k_2 = hf(x_n + \frac{1}{2}h, y_n + \frac{1}{2}k_1, z_n + \frac{1}{2}k_2)$$

$$k_3 = hf(x_n + \frac{1}{2}h, y_n + \frac{1}{2}k_2, z_n + \frac{1}{2}k_2)$$

$$k_4 = hg(x_{n+1}, y_n + k_3, z_n + k_3)$$

And

$$k_1 = hg(x_n, y_n, z_n)$$

$$k_2 = hg(x_n + \frac{1}{2}h, y_n + \frac{1}{2}k_1, z_n + \frac{1}{2}k_1)$$

$$k_3 = hg(x_n + \frac{1}{2}h, y_n + \frac{1}{2}k_1, z_n + \frac{1}{2}k_2)$$

$$k_4 = hg(x_{n+1}, y_n + k_3, z_n + k_3)$$

Hence, therefore the error (local) involved in the determination of Y_{n+1} from Y_o and Z_{n+1} from z_o is $O(h^5)$ as with Runge-Kutta method. (Jeffery and Dai, 2006 in Nwagor 2019)

Materials and Methods

In this study, if m represents a specified model parameter value that contributes to growth of targets cells, the growth of infected cells, and the growth of the viral load of the virions. We have utilized the following Matlab algorithm using a random function of order ten (10) having the following structure: $v = m+0.01$ times and(10) with this random generation function we are able to prove that the following that the following model parameter values of

$$T_{max} = 1300, \beta = 0.0002, \\ \delta = 0.5, \rho = 1000, v = 120, \\ c = 10, d = 0.01, I = 50, \alpha = 6.8$$

clearly satisfy a mathematical upper bound definition. This key assumption makes sense while the Port Harcourt HIV public health data should be considered as the best, minimum values in the context of modeling the characterization of a time dependent viral load of the virions.

Results

In The results of this study are presented as shown in Tables 1-10.

Table 1: Parameter selection using Random generation function indexed by Matlab algorithm of order 10; scenario 1.

Eg	Parameter	Parameter value	Random Parameter Value
1	d	0.01	0.016238553056
2	δ	0.5	0.504918262176
3	c	10	10.00629268117
4	α	6.8	6.803981847923
5	T_{max}	1300	1340.752840693
6	s	5	5.005399158708
7	β	0.0002	0.007369622933
8	ρ	1000	1056.182770771

Table 2: Parameter selection using Random generation function indexed by Matlab algorithm of order 10; scenario 2.

Eg	Parameter	Parameter value	Random Parameter Value
1	d	0.01	0.014878049132
2	δ	0.5	0.504778347511
3	c	10	10.00414556370
4	α	6.8	6.803993470257
5	T_{max}	1300	1353.616176786
6	s	5	5.005590781608
7	β	0.0002	0.007482744160
8	ρ	1000	1061.587363677

Table 3: Parameter selection using Random generation function indexed by Matlab algorithm of order 10; scenario 3.

Eg	Parameter	Parameter value	Random Parameter Value
1	d	0.01	0.015783748452
2	δ	0.5	0.503845233884

3	c	10	10.00471427218
4	α	6.8	6.805236992209
5	T_{max}	1300	1355.180788041
6	s	5	5.005423792901
7	β	0.0002	0.004007139965
8	ρ	1000	1093.825987542

Table 4: Parameter selection using Random generation function indexed by Matlab algorithm of order 10; scenario 4.

Eg	Parameter	Parameter value	Random Parameter Value
1	d	0.01	0.016279884229
2	δ	0.5	0.505121899926
3	c	10	10.00510851985
4	α	6.8	6.806035959270
5	T_{max}	1300	1363.881851207
6	s	5	5.005766871935
7	β	0.0002	0.005100675264
8	ρ	1000	1039.122809121

Table 5: Parameter selection using Random generation function indexed by Matlab algorithm of order 10; scenario 5.

Eg	Parameter	Parameter value	Random Parameter Value
1	d	0.01	0.015324483187
2	δ	0.5	0.504625210898
3	c	10	10.00363266988
4	α	6.8	6.804740920781
5	T_{max}	1300	1368.177281911
6	s	5	5.006936303499
7	β	0.0002	0.005931664864
8	ρ	1000	1085.199735721

Table 6: Parameter selection using Random generation function indexed by Matlab algorithm of order 10; scenario 6.

Eg	Parameter	Parameter value	Random Parameter Value
1	d	0.01	0.015324483187
2	δ	0.5	0.504625210898
3	c	10	10.00363266988
4	α	6.8	6.804740920781
5	T_{max}	1300	1368.177281911
6	s	5	5.006936303499
7	β	0.0002	0.005931664864
8	ρ	1000	1085.199735721

Table 7: Parameter selection using Random generation function indexed by Matlab algorithm of order 10; scenario 7.

Eg	Parameter	Parameter value	Random Parameter Value
1	d	0.01	0.015104488382
2	δ	0.5	0.504287331791
3	c	10	10.00709667600
4	α	6.8	6.804872763926
5	T_{max}	1300	1345.060622342
6	s	5	5.006164437469
7	β	0.0002	0.005851473248
8	ρ	1000	1029.343663474

Table 8: Parameter selection using Random generation function indexed by Matlab algorithm of order 10; scenario 8.

Eg	Parameter	Parameter value	Random Parameter Value
1	d	0.01	0.014343716774
2	δ	0.5	0.504289171699
3	c	10	10.00465983788
4	α	6.8	6.804262108046
5	T_{max}	1300	1344.425522022
6	s	5	5.005642880654
7	β	0.0002	0.005473811254
8	ρ	1000	1097.530736270

Table 9: Parameter selection using Random generation function indexed by Matlab algorithm of order 10; scenario 9.

Eg	Parameter	Parameter value	Random Parameter Value
1	d	0.01	0.012804635511
2	δ	0.5	0.000504318283
3	c	10	10.00648045724
4	α	6.8	6.803993868287
5	T_{max}	1300	1345.780558517
6	s	5	5.006202005058
7	β	0.0002	0.004443281469
8	ρ	1000	1090.546802208

Table 10: Parameter selection using Random generation function indexed by Matlab algorithm of order 10; scenario 10.

Eg	Parameter	Parameter value	Random Parameter Value
1	d	0.01	0.013646065550
2	δ	0.5	0.505131926954
3	c	10	10.00570463411
4	α	6.8	6.804496477385
5	T_{max}	1300	1347.988191407
6	s	5	5.005020048792
7	β	0.0002	0.003759058161
8	ρ	1000	1038.028715741

Discussion of Results

From Tables 1-10, it is observed that each model parameter value is strictly bounded above by its least upper bound which clearly shows that the choice of the model parameter values is the minimum for each range such as $0 < m < \text{random parameter value}$, where m specifies a model parameter value. This observation is consistent for Tables 1-10. However, results show that the value of d ranges from 0.012804635511 to 0.016279884229. The parameter value for δ ranges from 0.000504318283 to 0.505131926954, whereas the parameter value for c ranging from 10.00363266988 to 10.00709667600. More so, the parameter value of α lie between 6.803981847923 and 6.806035959270. The parameter value T_{max} ranging 1340.752840693 to 1368.177281911, while the value of s lie between 5.005020048792 and 5.006936303499. The β values ranges from 0.003759058161 to 0.007482744160 whereas the parameter value for ρ has a minimum of 1029.343663474 and a maximum of 1097.530736270.

Conclusion and Further Research

This study work have successfully utilized a Matlab function algorithm to conduct a systematic random generation of HIV infection model parameter values from which the choice of non-stochastic parameter values can be deduced as the minimum estimated values in this context. A future investigation will attempt to look at error analysis on the difference between pairs of such random generated data.

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