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An Example of Application of CBM to Intersection-Union Hypotheses Testing

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ABSTRACT

Application of CBM to the testing of the intersection of a sub-set of basic hypotheses against an alternative one is considered. Optimal decision rule allows us to restrict the Type-I and Type-II errors rates on the desired levels.

Keywords: Intersection-Union Hypotheses; Constrained Bayesian Method; Type-I and Type-II Error Rates

Introduction

In many applications, there arise the problems when basic or/and alternative hypotheses can be represented as the union or intersection of several sub-hypotheses Sen Gupta [1], Roy [2]. The general methodology of testing union-intersection hypotheses was offered in Roy [2]. An idea of this methodology consists in consideration in pairs of sub-hypotheses from basic and alternative hypotheses. Each pair of hypotheses is chosen one by one from basic and alternative hypotheses sub-sets accordingly. Final regions, for acceptance of basic and alternative hypotheses, are defined as intersection of the appropriate sub-regions. The reverse scenario, when basic and/or alternative hypotheses are presented as intersection of the appropriate sub-sets of hypotheses, is considered in Sen Gupta [1]. The special methodology, giving the powerful decision rule, based on Pivotal Parametric Product (P^3), is offered there. Application of constrained Bayesian method (CBM), developed by author Kachiashvili [3], to the considered type of hypotheses in one concrete case, is offered below. Here is shown that obtained decision rule allows us to restrict the Type-I and Type-II error rates on the desired level, in this case.

Statement and Solution of the Problem

Let a random variable X follow the distribution $f(x;\theta)$, where θ is scalar parameter. Let's consider testing

$$H_0: \theta \le \theta_1 \text{ or } \theta \ge \theta_2 \text{ vs } H_1: \theta_1 < \theta < \theta_2 . \quad (1)$$

Hypothesis H_0 can be represented as $\bigcup_{i=1}^{2} H_{0i}$, where the sub-hypotheses H_{01} and H_{02} are given by H_{01}: $\theta \le \theta_1$ and H_{02}: $\theta \ge \theta_2$ That means, we have to test basic hypothesis $H_0 \equiv \bigcup_{i=1}^{2} H_{0i}$ against alternative one H_1 when the condition $H_0 \bigcup H_1 = R^m$ is fulfilled.

Let's use one of possible statements of CBM, concretely Task 2 (Kachiashvili, 2018), for solving this problem. Then problem has the following form:

$$r_\delta = \min_{\{\Gamma_j\}} \left\{ K_1 \cdot \left[\int_{\Gamma_1} \left(p(H_{01}) p(x|H_{01}) + p(H_{02}) p(x|H_{02}) \right) dx + \right. \right.$$

$$r_\delta = \min_{\{r_j\}} \left\{ K_1 \cdot \left[\int_{r_1} \left(p(H_{01}) p(x/H_{01}) + p(H_{02}) p(x/H_{02}) \right) dx \right] \right\} \quad (2)$$

$$\left. \left. \int_{\Gamma_{01}} p(H_1) p(x|H_1) dx + \int_{\Gamma_{02}} p(H_{02}) p(x|H_{02}) dx \right] \right\}$$

subject to

$$\int_{\Gamma_{01}} p(x|H_{01}) dx \ge 1 - \frac{r_2^{01}}{K_0 \cdot p(H_{01})} ,$$

$$\int_{\Gamma_{02}} p(x|H_{02}) dx \ge 1 - \frac{r_2^{02}}{K_0 \cdot p(H_{02})} , \quad (3)$$

$$\int_{\Gamma_1} p(x|H_1) dx \ge 1 - \frac{r_2^{1}}{K_0 \cdot p(H_1)}$$

Here $p(H_i)$, $p(x \mid H_i)$, $i \in \Psi$, $\Psi \equiv \{01, 02, 1\}$ are a priory probability and the marginal density of x given H_i, Γ_i is hypothesis H_i acceptance region, r_2^i are the restriction levels, K_0 and K_1 are the losses for incorrectly rejection and incorrectly accepting of hypotheses, respectively.

The solution of the problem (2) and (3) by Lagrange method gives

$$\Gamma_{01} = \left\{ x : K_1 \cdot \left(p(H_{02} \mid x) + p(H_1 \mid x) \right) < K_0 \cdot \lambda_{01} \cdot p(H_{01} \mid x) \right\},$$

$$\Gamma_{02} = \left\{ x : K_1 \cdot \left(p(H_{01} \mid x) + p(H_1 \mid x) \right) < K_0 \cdot \lambda_{02} \cdot p(H_{02} \mid x) \right\}, \quad (4)$$

$$\Gamma_1 = \left\{ x : K_1 \cdot \left(p(H_{01} \mid x) + p(H_{02} \mid x) \right) < K_0 \cdot \lambda_1 \cdot p(H_1 \mid x) \right\}.$$

where Lagrange multipliers λ_{01}, λ_{01} and λ_{01} are determined so that in the conditions (3) equalities take place.

The errors of the Type-I and the Type-II, accordingly, are:

$$\alpha = \int_{\Gamma_1} p(x \mid H_{01}) dx + \int_{\Gamma_1} p(x \mid H_{02}) dx = p(x \in \Gamma_1 \mid H_{01}) + p(x \in \Gamma_1 \mid H_{02}),$$

$$\beta = \int_{\Gamma_{01}} p(x \mid H_1) dx + \int_{\Gamma_{02}} p(x \mid H_1) dx = p(x \in \Gamma_{01} \mid H_1) + p(x \in \Gamma_{02} \mid H_1). \quad (5)$$

Theorem 1. Testing hypotheses (1), using CBM 2 with restriction levels of (3) ensures a decision rule (4) with the error rates of the Type-I and Type-II, restricted by the following inequalities

$$\alpha \le \frac{r_2^{01}}{K_0 \cdot p(H_{01})} + \frac{r_2^{02}}{K_0 \cdot p(H_{02})},$$

$$\beta \le \frac{r_2^{1}}{K_0 \cdot p(H_1)}. \quad (6)$$

Proof. In contradistinction to the classical case, the following condition is fulfilled in CBM (Kachiashvili, 2018)

$$p(x \in \Gamma_{01} \mid H_i) + p(x \in \Gamma_{02} \mid H_i) + p(x \in \Gamma_1 \mid H_i) + p(imd \mid H_i) = 1, \quad i \in \Psi,$$

$\Psi \equiv \{01, 02, 1\}$.

Then the Type-I error rate can be rewritten as follows

$$\alpha = 1 - p(x \in \Gamma_{01} \mid H_{01}) - p(x \in \Gamma_{02} \mid H_{01}) - p(imd \mid H_{01}) +$$

$$+1 - p(x \in \Gamma_{01} \mid H_{02}) - p(x \in \Gamma_{02} \mid H_{02}) - p(imd \mid H_{02}) \le$$

$$\le \frac{r_2^{01}}{K_0 \cdot p(H_{01})} - p(x \in \Gamma_{02} \mid H_{01}) - p(imd \mid H_{01}) +$$

$$+ \frac{r_2^{02}}{K_0 \cdot p(H_{02})} - p(x \in \Gamma_{01} \mid H_{02}) - p(imd \mid H_{02}).$$

The correctness of the first inequality of (6) is obvious from here.

Similarly, for the Type-II error, we have

$$\beta = 1 - p(x \in \Gamma_1 \mid H_1) - p(imd \mid H_1) \le \frac{r_2^1}{K_0 \cdot p(H_1)} - p(imd \mid H_1).$$

And the correctness of the second inequality of (18) is obvious from here.

Conclusion

Application of CBM to one concrete example of intersection-union hypotheses testing is considered in the paper. Here is proved that, obtained decision rule guaranty the Type-I and Type-II error rates restricted on the desired levels.

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