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# Growth of Phytoplankton

**Udriste C* and Tevy I**

*Department of Mathematics-Informatics, Faculty of Applied Sciences, Romania*

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***Corresponding author:** Udriste C, Department of Mathematics-Informatics, Faculty of Applied Sciences, Splaiul Independentei 313, Bucharest, 060042, Centenary of Romanian Great Union 1818-2018, Romania

**Abstract**

A central problem in biological dynamical systems is to determine the boundaries of evolution. Although this is a general problem, we prefer to give solutions for growth of the phytoplankton. AMS Mathematical Classiftcation: 92D40, 35B36, 35Q92, 37N25.

**Keywords:** Phytoplankton Dynamical System; Phytoplankton Sub-strate; Phytoplankton Biomass; Intracellular Nutrient Per Biomasss

## Introduction

### Phytoplankton Dynamical System

In the paper of Bernard - Gouze [1] one analyse a model of phytoplankton growth based on the dynamical system

$$x^1(t) = 1 - x^1(t) - \frac{1}{4}x^1(t)x^2(t),$$

$$x^2(t) = 2x^2(t)x^3(t) - x^2(t),$$

$$x^3(t) = \frac{1}{4}x^1(t)\_2x^{3^2}(t),$$

where $x^1$ means the substrate, $x^2$ is the phytoplankton biomass and $x^3$ is the intracellular nutrient per biomass, with the physical domain $R^3 + : x = (x^1, x^2, x^3) \geq 0$. The previous dynΣamical system is non cooperative and has the equilibrium point $1,0,\frac{\Sigma\sqrt{1}}{2}$. We introduce the phytoplankton vector field $X = (X^1, X^2, X^3)$ of components $X^1 = 1 - x^1 - \frac{1}{4}x^1x^2,\ X^2 = 2x^2x^3 - x^2,\ X^3 = 1x^1_4 - 2x^{3^2}1$ and the maximal field line x = x(t, x0), t ∈ I, which satisfies the initial condition x(t0, x0) = x0. In order to find bounds for substrate, biomass, and intracellular nutrient per biomass, we use the techniques of optimization developed in our papers [2-6].

### Bounds for Phytoplankton Substrate

We use the following problem: ftnd max f ($x_1$, $x_2$, $x_3$) = x1 with the restriction $x = x(t, x_0)$. We set the critical point condition $(\nabla f, X) = 0$. In this case $\nabla f = (1,0,0)$. It follows the relation $1 - x^1 - \frac{1}{4}x^1x^2 = 0$. The convenient solution (critical point) $\overline{x} = (\overline{x}^1, \overline{x}^2, \overline{x}^3)$ must be

$$\overline{x} = ^{-} \overline{x}^1 \in [0,1), x^{-2} = \frac{4(1\_\overline{x}^1)}{\overline{x}^1}, \overline{x}^1 \geq 0\Sigma.$$

The sufficient condition Hess $f(X,X)(\overline{x}) + (\nabla f, D_X X)(\overline{x}) < 0$ reduces to $(\nabla f, D_X X)(\overline{x}) < 0$ or $1 + \frac{\overline{x}^{-2\Sigma-}}{4} 1 - \overline{x}^1 - \frac{1}{4}\overline{x}\,\overline{x}^{-1-2\Sigma} + \frac{\overline{x}^1}{4}(2x^2x^3 - \overline{x}^2) > 0.$

Since at the critical point we have $\overline{x}^2 = \frac{4(1\_x^{-1})}{x^{-1}}$, the condition goes to $(1 - x^{-1})(2x^{-3} - 1) > 0$, and the convenient condition is $\overline{x}^1 < 1, 2\overline{x}^3 > 1$. Theorem 2.1. Suppose that on a evolution line (field line) it exists a point x¯ at which we have

$$\overline{x}^1 < 1, 2\overline{x}^3 > 1, \overline{x}^2 = \frac{4(1\_\overline{x}^1)}{\overline{x}^1}$$

Then the phytoplankton substrate has an upper bound at this point.

### Bounds for Phytoplankton Biomass

Let us use the problem: ftnd max $g(x^1, x^2, x^3) = x^2$ subject to $x = x(t, x_0)$.

**Growth of phytoplankton**: The critical point condition is $(\nabla g, X) = 0$. Since $\nabla g = (0,1,0)$, it follows the relation $2x^2x^3x = 0$. The convenient solution (critical point) $\overline{x}$ = $(\overline{x}^1, \overline{x}^2, \overline{x}^3)$ must have $\overline{x}^3 = 1$. The sufficient condition Hess $g(X,X)(\overline{X}) + (\nabla g, D_X X)(\overline{X}) < 0$ reduces to $(\nabla g, D_X X) < 0$ or $\overline{x}^2(2\overline{x}^3 - 1)^2 + 2\overline{x}^2(1\frac{\overline{x}^1}{4} - 2\overline{x}^3)^2 < 0$. Since at the critical point we have , the sufficient condition leads to $\overline{x}^2(\overline{x}^1 - 2) < 0, i.e., \overline{x}^1 - 2 < 0$.

**Theorem:** Suppose that on a evolution line (field line) it exists a point at which we have

$$\overline{x}^1 < 2, \overline{x}^2 > 0, \overline{x}^3 = \frac{1}{2},$$

then the $x^2(t)$ component of the corresponding field line has an upper bound at this point.

In the direct alternative, we build the composite function g(x(t, x0)). The condition

$$\frac{d}{dt} g(x(t,x_0)) = \frac{\partial .g}{\partial x}(x(t,x_0))\frac{dx^i}{dt}(t) = 0$$

reduces to $\frac{dx^2}{dt}(t) = 0, i.e., 2x^2x^3 - x^2 = 0$. the convenient solution is $x^3 = \frac{1}{2}$. The condition

$$\frac{d^2}{dt^2} g(x(t,x_0)) = \frac{\partial^2 .g}{\partial x^i \partial x^j}(x(t,x_0))\frac{dx^i}{dt}(t) + \frac{dx^j}{dt}(t) + \frac{\partial^2 .g}{\partial x^i}(x(t,x_0))\frac{d_2 x^j}{dt^2}(t) < 0,$$

becomes $2x^2(t)\frac{dx^3}{dt}(t) < 0, i.e., x^2 > 0, \frac{1}{4}x^1 - 2x^{32} < 0.$. Replacing $x^3 = \frac{1}{2}$, we find $x^2 > 0, x^1 - 2 < 0$. The same result is obtained as in the previous method.

## Bounds for Intracellular Nutrient Per Biomass

Now the helping problem is: compute max $h(x^1, x^2, x^3) = x^3$ with the restriction $x = x(t, x_0)$.

The critical point condition is $(\nabla h, X) = 0$. Since Δh = $\nabla h = (0,0,1)$, it follows the relation $\frac{1}{4}x^1 - 2x^{32} = 0$. The critical point $\bar{x} = (\bar{x}^1, \bar{x}^2, \bar{x}^3)$ must be $\bar{x} = \bar{x}^1 \geq 0, \bar{x}^2 \geq 0,, \bar{x}^3 = \frac{\sqrt{\bar{x}^1}\ \Sigma}{2\sqrt{2}}$. The sufficient condition Hess $h(X,X)(\bar{x}) + (\nabla h, D_X X)(\bar{x}) < 0$ reduces to

$$(\nabla h, D_X X)(\bar{x}) < 0 \text{ or } \frac{1}{4}1 - \bar{x}^1 - \frac{1}{4}\bar{x}^1\bar{x}^2 - 4\bar{x}^3\frac{1}{4}\bar{x}^1 - 2\overline{x^3}^2 < 0$$

Since at the critical point we have $\bar{x}^3 = \frac{\sqrt{\bar{x}^1}}{2\sqrt{2}}$ the condition goes to $1 - \bar{x}^1 - \frac{1}{4}\bar{x}^1\bar{x}^2 < 0, i.e., 1 < \bar{x}^1(1 + \frac{1}{4}\bar{x}^2)$

**Theorem:** Suppose that on a evolution line (field line) it exists a point

$\bar{x}$ at which we have $1 < \bar{x}^1(1 + \frac{1}{4}\bar{x}^2), \bar{x}^3 = \frac{\sqrt{\bar{x}^1}}{2\sqrt{2}}$. Then the intracellular nutrient per biomass has an upper bound at this point.

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