BIOMEDICAL
Journal of Scientific & Technical Research

Case Report

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A Hierarchical Reduced Markov Model for Reliability Evaluation of Phased-Mission Systems

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Abstract

This paper presents a reduced Markov model to evaluate the Phased-Mission System (PMS) reliability. The time cost and storage requirement are very huge using traditional Markov model to analyze the reliability of PMS as the number of components increases to a large scale. The hierarchical reduction approach proposed in this paper can account for the PMS which is composed of different level components. Such as thrust equipment of an aircraft can be further decomposed into compressor and turbine runner. The reduced Markov model by hierarchical reduction will have a smaller number of system states than the traditional model. Finally, the effectiveness and correctness of our approach is analyzed by comparing with the traditional Markov method.

Nomenclature

Introduction

Many practical systems, such as those encountered in electronic, aerospace and nuclear power have multiple phases of operations during their mission and are generally referred to as Phased-Mission System (PMS) [1]. Compared with general system, the reliability evaluation of PMS is much more complicated because of the components dynamic behavior as well as their dependence across the phases. Considerable efforts have been made in the analysis of PMS. Generally, existing methods can be roughly classified into two groups: Analytical methods and simulation methods [2]. The analytical methods are further classified into combinatorial approaches, state-based approaches, hierarchical approaches and modularization approaches. The combinatorial approaches, especially the binary decision diagram-based algorithm [3,4], exploit the Boolean algebra to reduce the model scale, which are effective in evaluating non-repairable PMS. The state-based approaches are mainly based on Markov model [5,6], which suffers from the state-explosion problem when is applied to large-scale systems. The simulation methods typically offer greater generality in system representation, but they suffer from computational inefficiency and great computer memory consumption [7].

The above-mentioned methods mainly apply to small-scale PMSs due to the high computational complexity. This paper proposed a reduced Markov reliability model by using hierarchical reduction techniques. Compared with the traditional Markov model, the proposed method can efficiently reduce the number of system states, and avoid the well-known state-explosion problem. The remainder of the paper is organized as follows. Section 2 describes the hierarchical reduced Markov model. Section 3 illustrates our method with a specific example. Lastly, section 4 concludes the study and discusses future works.

The Hierarchical Reduced Markov Model

It is common that the real engineering systems provide hierarchical feature which can be used to advantage in the PMS reliability analysis. For example, thrust subsystem for an aircraft frequently effect all phases of a mission, maybe in different quantities, at each stage of flight [8]. Furthermore, the top-level subsystem can be decomposed into lower level components such as compressor and turbine runner. Such structure is common in many other engineering systems, not just those relating to aircraft. The hierarchical architecture is also a usual style in software reliability estimation, in which systems are decomposed into different layers according to the components' calling relations [9]. And it is also used to describe the interpreter relations of different layers in communication software. Accounting for hierarchical structure in the phase failure logic will enable a more reduced Markov model to be developed. One way of taking advantage of these features is using hierarchical reduction method which can reduce the state space of Markov model efficiently by subsystems replacement. The

main idea of this method is generating a hypothetical component to replace the top-level subsystem. The components of subsystems are considered to be all in series or in parallel. Thus, the failure rate and repair rate of the hypothetical component can be derived by manipulation of the corresponding components' parameters in the subsystem. Also, it is important that the subsystems replaced need to be totally independent of all other sections of the mission failure fault tree.

The detailed process of the hierarchical reduction method is discussed taking a n components subsystem for example. Let λ_i and μ_i be the failure rates and repair rates of component C_i. Also, λ_s and μ_s are the failure rate and repair rate of hypothetical component C_s representing the subsystem.

1) if the n components are series, λ_s can be given as

$$\lambda_S = \sum_{i=1}^{n} \lambda_i \tag{1}$$

Let A_s denote availability of the subsystem, where   is defined by

$$A_S = \frac{\mu_S}{\lambda_S + \mu_S} \tag{2}$$

A series system is working when all of the components do not fail. If state 1 represents that all components are operational and A_1 is the steady-state availability of system in state 1, then A_1 can be computed as follows.

$$A_1 = 1 \Big/ (1 + \sum_{i=1}^{n} \frac{\lambda_i}{\mu_i}) \tag{3}$$

Since state 1 is the only working state in series system, it can be derived that. $A_s = A_1$ Moreover, according to Eq. (2) and (3), we have

$$\mu_S = \sum_{i=1}^{n} \lambda_i \Big/ \sum_{i=1}^{n} \frac{\lambda_i}{\mu_i} \tag{4}$$

From Eq. (1) and (4), we can use the hypothetical component C_s with failure rate λ_s and repair rate μ_s to replace the series subsystem.

2) if the n components are parallel, μ_s can be given as

$$\mu_S = \sum_{i=1}^{n} \mu_i \tag{5}$$

For parallel system, the formula of availability A_s is presented in [19] as follows.

$$A_S = 1 - \prod_{i=1}^{n} (1 - A_i) \tag{6}$$

in which $A_i = \mu_i / (\lambda_i + \mu_i)$ is the availability of component c_i. Hence, using Eq. (2), (5) and (6), we obtain

$$\lambda_S = \sum_{i=1}^{n} \mu_i \cdot \left(\prod_{i=1}^{n} (\frac{\lambda_i}{\lambda_i + \mu_i}) \right) \Big/ \left(1 - \prod_{i=1}^{n} (\frac{\lambda_i}{\lambda_i + \mu_i}) \right) \tag{7}$$

From Eq. (5) and (7), we can use the hypothetical component c_s with failure rate λ_s and repair rate μ_s to replace the parallel subsystem.

Case Study

In this section we describe a case study to verify the proposed reduced Markov model for PMS reliability analysis. The mission reliability of the PMS has been calculated by the traditional Markov method. The example PMS is a space flight Telemetry, Tracking and Control (TT&C) system. The failure fault trees of the example TT&C mission are shown in Figure 1, in which station1, station2, and station3 are composed of C1~C6, C7~C8, and C9~C10 respectively. Assume that the failure rates and repair rates of components C1~C10 are unchanged across phases. The relevant parameters are detailed as follows: failure rates $\lambda_i = 3.33 \times 10^{-3}$; repair rates $\mu_i = 3.33 \times 10^{-2}$; phase transition time $t_1 = 20s$, $t_2 = 120s$ and $t_3 = 85s$.
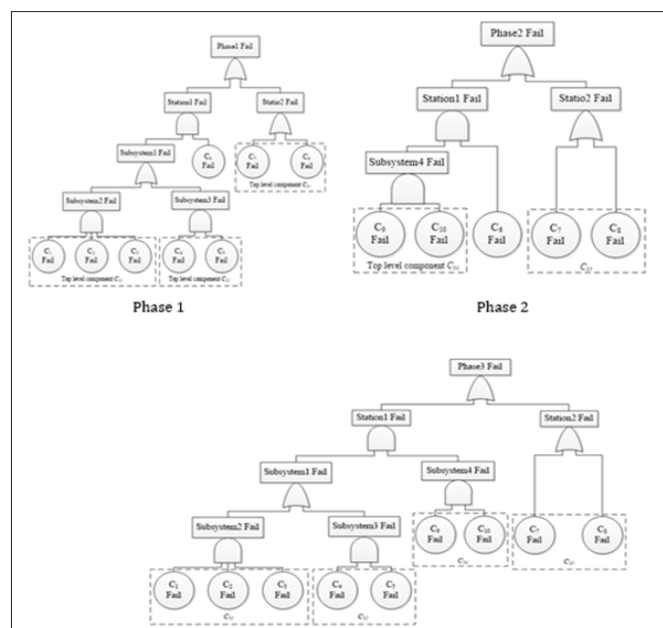

Figure 1: The failure fault tree of the three basic phases.

By analyzing fault trees of the three basic phases, there are four subsystems can be reduce using hierarchical reduction method. These are S_1 with components c_1, c_2 and c_3, S_2 with components c_4 and c_5, S_3 with components c_7 and c_8, and S_4 with components c_9 and c_{10}. Therefore, the top level subsystems S_1, S_2, S_3 and S_4 can be replaced by four hypothetical components c_{S1}, c_{S2}, c_{S3} and c_{S4}

respectively. For the series subsystem, failure rate and repair rate of the top-level components can be computed by using Eq. (1) and (4). Similarly, Eq. (5) and (7) can be used for the parallel subsystem. The computed parameters of the top-level hypothetical components are presented in Table 1. After hierarchical reduction, the new PMS phases configuration is shown in Figure 2.

Table 1: Computed parameters of the hypothetical components (unit: min).

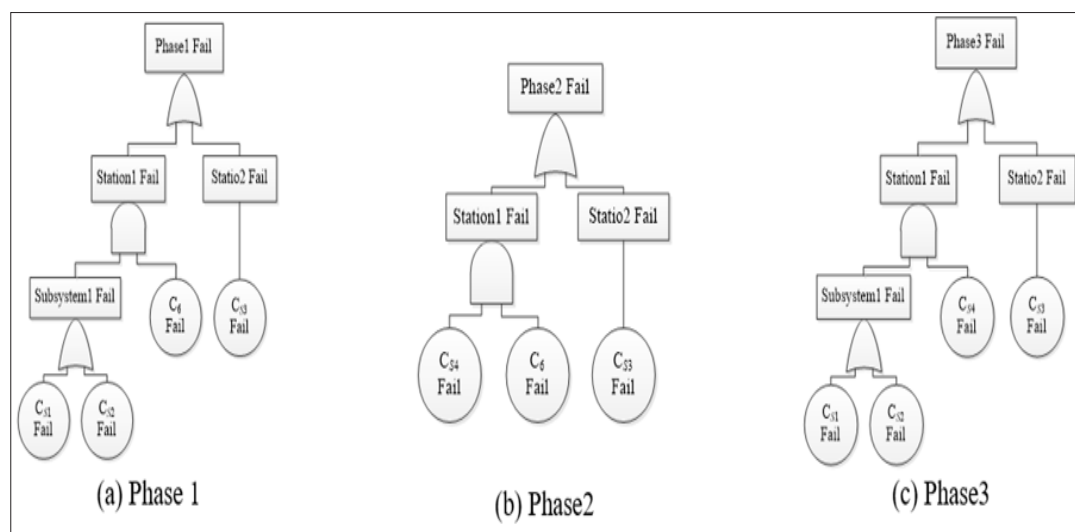| Parameters	c_{S1}	c_{S2}	c_{S3}	c_{S4}
Failure rates	$\lambda_{S1} = 7.519 \times 10^{-5}$	$\lambda_{S1} = 5.556 \times 10^{-4}$	$\lambda_{S3} = 6.667 \times 10^{-3}$	$\lambda_{S1} = 5.556 \times 10^{-4}$
Repair rates μ_i	$\mu_{S1} = 0.1$	$\mu_{S2} = 6.667 \times 10^{-2}$	$_{S3} = 3.333 \times 10$	$\mu_{S2} = 6.667 \times 10^{-2} 10$

Figure 2: New phases configuration of the PMS after hierarchical reduction.

From Figure 2, we can derive that the number of reduced system states of the three phases are 16, 8, and 16 respectively. For the original PMS, these are 256, 32, and 512 respectively. Because the number of components is decreased efficiently by using hypothetical components to replace the top-level subsystems, and it will slow down the increase in transition rate matrix scale. The PMS reliability and computation time of the traditional Markov method, and the hierarchical reduction method are presented in Table 2. The computing errors of each phase between traditional method and the hierarchical method are shown in Figure 3. Results in Table

2 indicate that there is an excellent match between the proposed method to the traditional Markov method, with relative error less than 1.19×10-5. Furthermore, the computation time of our method is less than the traditional Markov method. This is mainly because that the number of system states of simplified Markov model is less than that of traditional models. However, from Figure 3 we can see that the computing errors increase dramatically with the mission time. Thus, the hierarchical reduced Markov model is applicable for short duration PMS.

Table 2: Reliability and computation time of the example PMS.

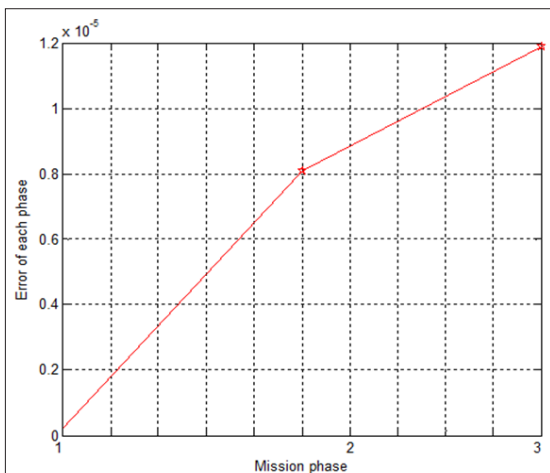|	Traditional Markov method		Proposed method	
	PMS reliability	**Time cost(s)**	**PMS reliability**	**Time cost(s)**
Phase 1	0.997780243	0.015	0.99778001	0.0011
Phase 2	0.984564486	0.063	0.98455638	0.0021
Phase 3	0.975309577	0.320	0.97529770	0.0020

Figure 3: Computing errors between traditional method and the hierarchical method.

Conclusion

This paper presents a reduced Markov model using hierarchical reduction approach to evaluate the PMS reliability. The traditional Markov model suffers from the problem of huge transition rate matrix. Our approach takes advantage of PMS provide hierarchical feature, and an simplify the original Markov model of each phase by hierarchical reduction. In comparison to the traditional Markov approach, the proposed method achieves less time cost, also has good computation accuracy. However, the hierarchical process may cause computation errors. In future work, more attention will be addressed on error analysis, and efforts should be made to improve current method to achieve more accuracy and efficiency.

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